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West Point, New York 10996

Hybrid Value-Utility Decision Analysis

OPERATIONS RESEARCH CENTER OF
EXCELLENCE TECHNICAL REPORT DSE-TR-02-02
DTIC #: TBD

Colonel Bill Klimack, Ph.D.
Director, Operations Research Center of Excellence

Approved by
Colonel Michael L. McGinnis, Ph.D.
Professor and Head, Department of Systems Engineering

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14. ABSTRACT

The Army faces complex decisions containing many variables and sources of uncertainty. Under standard DA approaches, value functions are employed under conditions of certainty and utility functions under conditions of uncertainty. Utility functions are unique to an individual, and so must be elicited from the decision maker. The time of military decision makers is extremely valuable, and makes employment of the utility approach problematic.

The concept of a hybrid value-utility DA model is promulgated. Such a model may be constructed beginning with a value model based on elicitations from subject matter experts, e.g., staff officers. Selected single dimensional value functions are replaced with the corresponding utility functions to form a hybrid model. Substitutions are prioritized through response surface methodology applied to the value functions of the decision model. Substitutions continue until the performance of the hybrid suitably estimates the performance of the true utility model. Construction of such a hybrid model minimizes the burden placed on the decision maker while maintaining the benefits of DA. A sample case illustrates the elicitation efficiency where an automatic target recognition classification system choice attribute set was reduced from 23 to eight.

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Abstract

The Army faces complex decisions containing many variables and sources of uncertainty. Decision Analysis (DA) provides a discipline that permits tools of quantitative reasoning to be employed to directly assist the decision maker. Under standard DA approaches, value functions are employed under conditions of certainty and utility functions under conditions of uncertainty. Utility functions are unique to an individual, and so must be elicited from the decision maker. The time of military decision makers is extremely valuable, and makes employment of the utility approach problematic.

The concept of a hybrid value-utility DA model is promulgated. Such a model may be constructed beginning with a value model based on elicitations from subject matter experts, e.g., staff officers. Selected single dimensional value functions are replaced with the corresponding utility functions to form a hybrid model. Substitutions are prioritized through response surface methodology applied to the value functions of the decision model. Substitutions continue until the performance of the hybrid suitably estimates the performance of the true utility model. Construction of such a hybrid model minimizes the burden placed on the decision maker while maintaining the benefits of DA.

A sample case illustrates the elicitation efficiency where an automatic target recognition classification system choice attribute set was reduced from 23 to eight. This reduction greatly improves the likelihood of senior leadership employment of decision analysis while providing a requisite decision model, improving military decision-making.

About the Author

Colonel William K. Klimack was a distinguished military graduate from Lehigh University, Pennsylvania, with a Bachelor of Science degree in Chemical Engineering in 1979. He earned a Master of Science in Applied Mathematics from the Johns Hopkins University, Baltimore, Maryland, in 1990 and a Master of Military Arts and Science from the United States Army Command and General Staff College, Fort Leavenworth, Kansas, in 1991. In 2002 he completed a Ph.D. in Operations Research at the Air Force Institute of Technology. He is a member of the Tau Beta Pi and Omega Rho national honorary societies. As an infantry officer, he has served at every level from platoon to army and in five divisions.

Acknowledgements

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1 Introduction

1.1 Purpose

The purpose of this technical report is to provide the algorithm for adapting a value-based decision analysis model into a model that adequately represents the utility model in a way that is most efficient to the decision maker. This hybrid value-utility model has demonstrated efficiencies, and is applicable for military decision-making.

1.2 Operations Research and Decision Making

All Operations Research (OR) is concerned with making decisions. The introductory remarks in *Introduction to Operations Research* (Hillier and Lieberman, 2001: 3) opines "to be successful, OR must ... provide positive, understandable conclusions to the decision maker(s)." Decision analysis (DA) provides a framework for examining complex decisions made under conditions of uncertainty, particularly for single, non-repeatable situations. The DA literature continues to expand, testifying to its success. Besides extensions to theory, DA has found application in many fields.

Decision analysis provides a bridge between the objectivist and subjectivist formulations of probability, linking the decision maker's intrinsic attitudes to the external environment with mathematical tools. In a sense, DA has returned to the roots of probability theory, to the work of Pascal and Bernoulli. In providing this confluence, DA provides a solid analytical foundation for problems of increasing complexity.

Decision makers likely will face increasingly complex situations because of increasingly complex technologies, more rapid communications, faster management practices, and increasing demand for organizational efficiency. Frequently, decisions involve objectives that are in conflict. Acceptable alternatives are challenging to develop. Decisions often involve intangible aspects and are of interest to multiple groups with differing views and may involve multiple decision makers. Often decisions involve expertise from several unrelated disciplines. Uncertainty also has a significant impact on decisions. Important decisions typically involve high

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stakes, are complex in nature, are multi-disciplinary, and need to be defensible (Keeney, 1982: 803 – 806). All these aspects of decisions challenge the decision maker, and the operations researcher.

1.3 Multiple Objective Decision Analysis

Keeney (1982: 806) defines decision analysis as "a formalization of common sense for situations which are too complex for informal use of common sense." He also offers a more technical definition: "a philosophy, articulated by a set of logical axioms, and a methodology and collection of systematic procedures, based on those axioms, for responsibly analyzing the complexities inherent in decision problems." The term *decision analysis* is sometimes used more broadly to refer to the analysis of any decision (see, for example, [Wallace, 2000]). In this document, the term decision analysis (DA) refers to the OR methodologies derived from this axiomatic basis, those that produce a preference function.

Multiple Objective Decision Analysis (MODA) is employed to permit the decision maker to clarify in her own mind which alternative she should select. The intent of Multiple Objective Decision Analysis is to be prescriptive, to provide what is the most prudent course available in a bewildering decision context, as established from examination of underlying preference components. Multiple objective decision analysis is not descriptive in that it does not attempt to replicate how an individual makes a decision. It is also not normative in that it accounts for deviation from preferences that would be expected of one who was strictly a formalist in application of mathematical principles. (Keeney and Raiffa, 1993: xv). (A less fastidious taxonomy of decision making considers the categories normative and prescriptive to be identical. For example, see Skinner [1999: 13].)

Operations research is credited as being the first organized scientific discipline devoted to decision-making. Emerging as a profession during the Second World War, operations research analysts played important roles in developing effective tactics for air and sea forces. Postbellum, those skilled in OR techniques found applications in the private sector and operations research continued to develop. The problems that were well suited for application of OR tools were repetitive in nature. Whether searching for enemy aircraft or scheduling production, OR techniques were found to be useful. Operations research flourished at the lower and middle levels of management. But executive decisions tend to be unique, not repetitive, involve large

portions of an organization's resources, characterized by high degrees of uncertainty, and typically involve the decision maker's personal risk attitudes. Operations research techniques were found lacking for application in the boardroom. (Matheson and Howard, 1968: 21 – 22)

Decision analysis provides a number of advantages. Besides extending analysis beyond repetitive decision situations, it codifies the process of making the actual decision. Prior to the development of DA, many man-months of effort were devoted to analysis of particular problems. This analysis may have been development and employment of a simulation, the application of statistical tools to historical or experimental data, or similar efforts. The analytic results were reduced to graphs, tables, or other conveyances and presented to the decision maker. The decision maker then was forced to absorb the information, process it in some internal, usually unspecified manner, and then announce his decision. Decision analysis permits the tools of operations research to be brought to bear on the decision itself to assist the decision maker. Also, the decision analytic process itself has inherent advantages. The DA process forces the decision maker to think hard about her preferences. While this requires an investment of time, generally participants indicate that the exercise was beneficial in making them devote the effort to gain insight into the problem. (For example, see Felli, Kochevar, and Gritton, 2000: 60 – 61). Corner and Kirkwood (1991: 206) observe, "Decision analysis provides tools for quantitatively analyzing decisions with uncertainty and/or multiple conflicting objectives. These tools are especially useful when there is limited directly relevant data so that expert judgment plays a significant role." Kirkwood (1992: 37) notes that DA's ability to perform tradeoffs among multiple objectives makes it valuable for government decision making. Within decision analysis, multiattribute utility theory is considered the leading methodology. Keeney and Raiffa (1993: xi) state, "Decision analysis is widely recognized as a sound prescriptive theory. When a decision involves multiple objectives – and this is almost always the case with important problems – multiattribute utility theory forms the basic foundation for applying decision analysis."

Decision analysis is widely applicable to military decision-making. Buede and Bresnick (1992) show that decision analysis is useful in the materiel acquisition process. Vendor engineers viewed value models distributed with the request for proposal for the US Marine Corps (USMC) Mobile Protected Weapon System to industry as helpful. The USMC found that all ten conceptual responses to the request for proposal were "outstanding" (115). Later work for the

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USMC involved adapting this value model for off-the-shelf procurement of the Light Armored Vehicle. A utility model was employed to determine the best air defense system mix for a US Army heavy division. The authors used a utility model to assess whether control of the Patriot Air Defense System should be transferred from the US Army to the US Air Force. A separate study group conducted the effort in parallel with a simulation approach. The two efforts were in concordance, the DA results were more easily interpreted and more defensible, and the simulation was an order of magnitude more expensive and slower (121). The authors include a list of 26 other materiel acquisition projects that have successfully employed decision analysis. Other notable military decision analytic efforts are SPACECAST 2020 (Burk and Parnell, 1997) and AIR FORCE 2025 (Jackson, 1996).

Clemen (2001: 6 – 8) describes the decision analysis methodology as a five-step process. The initial step is to identify the decision situation and articulate objectives. The second step is to identify alternatives. Next the problem is decomposed and modeled. Three aspects are modeled: the problem structure, the uncertainty, and the preferences. The structure of the problem is typically modeled using decision trees and influence diagrams. Concepts of probability theory are employed to capture aleatory aspects of the decision problem. Preferences are represented with value or utility functions. This step, according to Keeney (1982: 813), is unique to the discipline of decision analysis. Next, the modeling effort is used to analyze the decision and the best alternative is chosen. A sensitivity analysis is typically performed as the last step to determine the robustness of a solution. This process is iterative, and repeated until an adequate solution is determined. Figure 1 shows the process graphically. (Alternate, but similar, constructs of the decision analysis methodology are available in Skinner [1999: 15] and Keeney [1982: 807 – 820].)

1.4 Benefits of the Hybrid Value-Utility Approach

In complex problems the number of attributes to be examined is large. For example, the United States Air Force 2025 Study, which was commissioned to develop a recommendation for allocation of that service's funds and organizational efforts towards development of key system concepts and their enabling technologies, had 134 attributes (Jackson, 1996; and Parnell, Conley, Jackson, Lehmkuhl, and Andrew, 1998). (An effort for the National Reconnaissance Office had

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┌─────────────────────────┐
│ Identify the decision situation │◄──────────┐
│ and understand objectives.     │          │
└─────────────────────────┘          │
            │                        │
            ▼                        │
    ┌─────────────────┐              │
    │ Identify alternatives. │◄──────────┤
    └─────────────────┘              │
            │                        │
            ▼                        │
┌───────────────────────────┐        │
│ Decompose and model the problem: │◄─────┤
│ Model of problem structure.      │      │
│ Model of uncertainty.            │      │
│ Model of preferences.            │      │
└───────────────────────────┘        │
            │                        │
            ▼                        │
    ┌─────────────────────┐          │
    │ Choose the best alternative. │     │
    └─────────────────────┘          │
            │                        │
            ▼                        │
    ┌─────────────────┐              │
    │ Sensitivity analysis │              │
    └─────────────────┘              │
            │                  Yes   │
            ▼                        │
      ◇ Is further analysis needed? ◇────┘
            │
           No
            ▼
┌───────────────────────────┐
│ Implement the chosen alternative. │
└───────────────────────────┘
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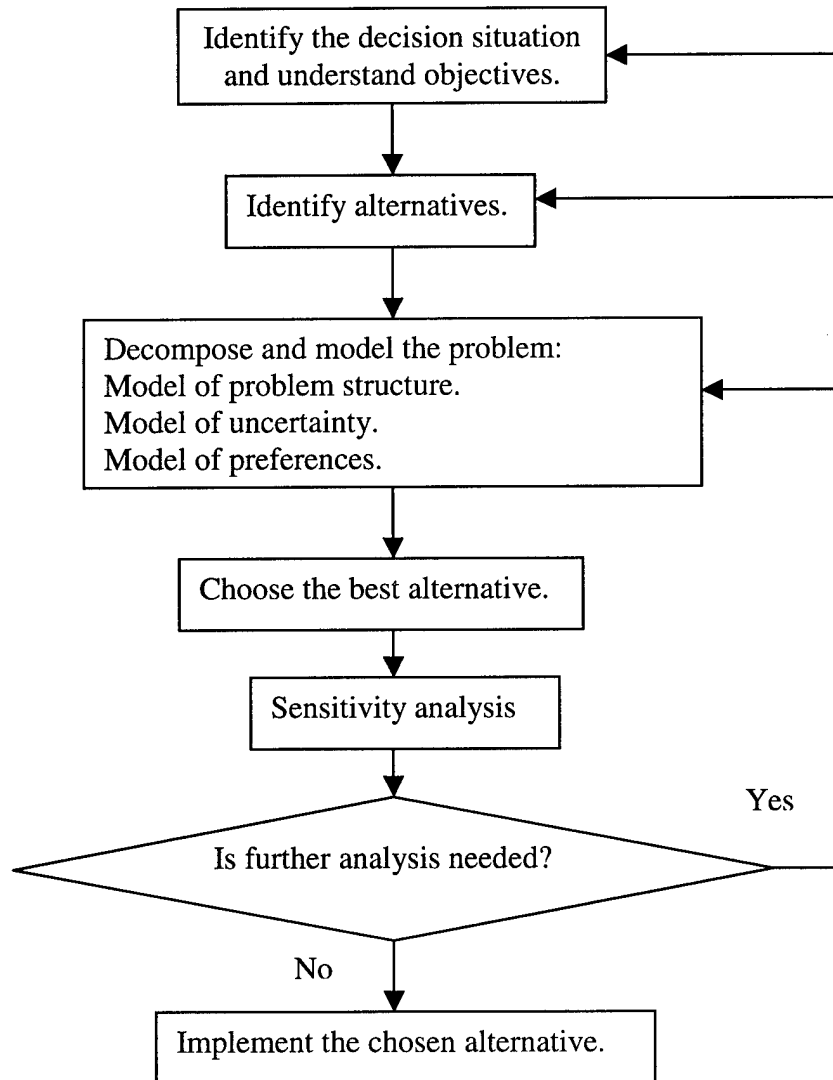
Figure 1. The Decision Analysis Process (after Clemen, 2001: 6).

69 evaluation measures [Parnell, 2001].) The attributes ranged from operational performance issues to technical feasibility of untested technology. Clearly such large sets of attributes, which draw on expertise from multiple disciplines, is beyond the ability of a single individual to make informed preference statements. Instead, the judgment of subject matter experts must be coalesced and presented to the decision maker. Under conditions of uncertainty, common for challenging decision situations, utility functions must be elicited personally from the decision maker. Such an approach is problematic, as for large, important decisions; the decision maker faces many competing demands upon her time. A methodology that permits the value model to be adapted to represent the decision maker's utility model with the least decision maker

interaction would be extremely valuable and critical for consideration of decision analysis in such cases.

1.5 Sequence of Presentation

The balance of the report is divided into five chapters. Chapter 2 will review the literature regarding value and utility functions in decision analysis and response surface methodology. Chapter 3 provides the theory supporting the hybrid value-utility algorithm. Chapter 4 presents the hybrid value-utility algorithm and provides an example. Chapter 5 provides an example application for Automatic Target Recognition Classification System selection and Chapter 6 briefly summarizes the report.

2 Literature Review

2.1 General

As discussed in Chapter 1, decision analysis is of recent development in operations research. However, the concepts involved date to the Enlightenment, when mathematical tools were brought to bear on decision-making. That work provided a foundation upon which modern economics rests, and within the last few decades has proven to be the inchoate form of decision analysis.

2.2 History of Utility and Value Theory

2.2.1 Pascal.

Hacking links the birth of scientific analysis of decision making to Pascal, and his well-known application of probability in what is now referred to as Pascal's Wager (Hacking, 1975: 11). Previous work with probability was limited to solving puzzles about games of chance. Here Pascal wrestles with the concept that God either is, or is not. If God exists, one should lead a pious life leading to salvation. If not, one may enjoy one's vices without regard to consequences after death. However the matter is unresolved until after death, and the choice of how to live must be made now. With the objective probability of God's existence unknown, Pascal focuses on the idea of the differing value of the potential outcomes, extending the ability to analyze uncertain situations beyond those posed about gaming tables. He is responsible for recognizing that the expected value of an uncertain (discrete) situation with n outcomes is equal to $\sum_1^n x_i p_i$, where x_i represents the potential outcomes and p_i their corresponding probabilities. Pascal vacillated on how to live his life, sojourning in a monastery, and then abandoning his refuge to the exhilaration of the casinos. His final choice, as salvation was preferable to damnation regardless of the probabilities involved, was the decision to return to the monastery for good. (Bernstein, 1996: 69 – 71).

2.2.2 Bernoulli.

The development of the concept of utility theory is credited to Daniel Bernoulli. In a 1738 paper published by the Imperial Academy of Sciences in St. Petersburg, Bernoulli states, "the value of an item must not be based on its price, but rather on the utility that it yields." (Bernoulli, 1738/1954: 24). He notes that under conditions of uncertainty, the expected value of an outcome is determined by multiplying the probabilities and the respective "gains." But he finds this inadequate for describing how individuals make decisions. He proposed that an individual would make a decision in order to maximize expected utility, rather than maximizing the expected amount of money (ducats, for Bernoulli's paper) or some other objective measure of worth. The calculation is made by determining the mathematical expectation employing utility in place of the units of currency, employed by previous researchers, yielding $\sum_{1}^{n} u(x_i) p_i$, where u is the utility function. He also asserted that for a given decision situation, different individuals would assign differing utilities to various possible outcomes.

Bernoulli extended his analysis by suggesting that an increase in utility resulting from a gain in wealth would be inversely proportional to the amount of previous possessions. This was another key insight. He provided a graph similar to Figure 2. This inverse relationship causes the concavity shown by the utility function. He also demonstrated that two individuals with identical utility functions, who engage in a wager equally likely to cause either to pay the other a fixed amount, were both facing a loss of expected utility. The concavity means that the increase in utility of winning is less than the reduction in utility of losing the same amount.

Bernoulli used the concept of utility to solve a question originally posed by his cousin, Nicolas Bernoulli in 1713. (With Daniel Bernoulli's solution, this problem has become known as the St. Petersburg Paradox.) Nicolas had developed a thought experiment where two individuals agree to a game between them with no house take. The first player will flip a coin repeatedly until the result is tails. Then he will pay the second player 2^n ducats (or dollars) where n is the number of coin flips. Before the game begins, a third player wishes to buy the right to play from the second. Nicolas notes that the expected value of the game in a monetary sense is infinite, as the payoff increases infinitely and the probability of achieving another heads never reaches zero. But most individuals would sell the game for some amount far less than infinity, say between 5 and 20 ducats (or dollars). Daniel Bernoulli's utility provides the answer.

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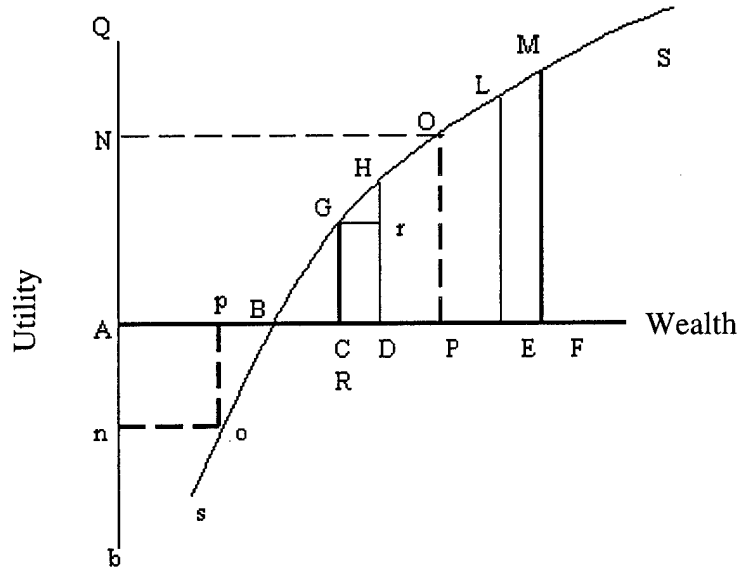
Figure 2. Bernoulli's Utility Curve. After Bernoulli (1738/1954).

The concavity of the utility function keeps the perceived worth of the gamble far from infinity. (Bernstein, 1996: 99 – 115).

2.2.3 Von Neumann-Morgenstern.

John von Neumann, in collaboration with Oskar Morgenstern, published the book *Theory of Games and Economic Behavior*. (Von Neumann and Morgenstern, 1953) They proposed an axiomatic basis for utility theory. These axioms provide the necessary and sufficient conditions for the existence of a utility function and are presented in Appendix A.

As summarized by McCord and de Neufville (1983b: 279), the axioms imply the existence of a cardinal utility function (sometimes referred to as a von Neumann-Morgenstern [vNM] function) that orders alternatives independent of probability distributions. Further, this function may be used to rank uncertain alternatives by taking the mathematical expectation of the vNM utility. That is, select the alternative with the highest expected utility as determined by,

$$U = \sum_{i=1}^{I} w_i E\left[u_i\left(x_i \right) \right] \tag{1}$$

for each alternative, where $i, (i = 1, 2, \cdots, I)$, is the index of evaluation measures, w_i are the relative weights, x_i is the level of the ith evaluation measure, and $u_i\left(x_i \right)$ are the corresponding single dimensional utility levels. Hence this theory is referred to as expected utility (EU) theory.

Despite some criticism of EU, Kimbrough (1994: 618) remarked, "For prescribing behavior, expected utility is still by far the most accepted theory." This position is shared by Edwards (1992: xi), who points out experts in the field at a 1992 conference "unanimously endorsed traditional SEU [subjective expected utility] as the appropriate normative model, and unanimously agreed that people don't act as the model requires." Clearly, expected utility is the preeminent model for non-behavioral aspects of decision making.

2.2.4 Measurable value functions

Farquhar and Keller (1989: 205) attribute the "strength of preference" concept to Pareto and Frisch. Stigler (1965) provides a history. Also known as "preference intensity," this is a quaternary relationship comparing preference differences. Measurable value functions were developed from an axiomatic basis as a means of comparing an individual's preferences among choices. (See Appendix B for the axioms.) In brief, given a set of outcomes $\{x_1, x_2, \cdots, x_n\}$ arranged in order of increasing preference, comparisons can be made between pairs of elements. For example, someone may be indifferent between the respective increase in preference between certain element pairs, $(x_i, x_j) \sim (x_k, x_l)$. This is read as the increase in preference from x_i to x_j is equal to the increase in preference from x_k to x_l. By establishing such relationships and normalizing the preferences over the unit interval (or any arbitrary interval), a measurable value function on an interval scale may be determined. Note that preference statements are made under conditions of certainty, and therefore do not involve the subject's risk attitudes. Fishburn (1967) provides a summary and classification of measurable value function methods. For a discussion of measurement theory (scale types), see Stevens (1946), Roberts (1979) or Krantz (1971). A brief summary is available in Forman and Gass (2001: 470).

2.3 Relationship of Value and Utility.

Opinion regarding the relationship between value and utility functions is varied. Von Winterfeldt and Edwards, (1986: 213) state that there is no significant distinction. They argue that there are no truly riskless situations, so decision-making under certainty is never actually encountered. Secondly, they argue that concavity associated with risk aversion may be explained by marginally decreasing return of value or through a regret expression. They also

argue that repetition reduces risk aversion and most choices are truly repetitive over a lifetime. Finally they opine that errors associated with elicitation of preference data exceed the distinctions between value and utility functions. McCord and de Neufville (1983a) provide an empirical study supporting this position.

Other authors disagree. Keeney states that value functions are used under conditions of certainty, and utility functions under conditions of uncertainty. He also opines that when measurable value functions and utility functions for a subject differ, this may indicate that the subject has a hidden objective (1992: 187). Bouyssou and Vansnick (1988: 110) observe, "Many authors have argued that they see no reason why lottery comparisons should coincide with preference difference comparisons." Clemen (1996: 553) argues that for decision analysis the distinctions are not significant for applications. Pennings and Smidts (2000) state that significant differences between utility functions and value functions were found by Krzysztofowicz (1983a and 1983b), Keller (1985b), Smidts (1997), and Weber and Milliman (1997). Their "results confirm the proposition that $u(x)$ and $v(x)$ are different constructs." (1343)

If value and utility functions are distinct, it is intuitively appealing that they are related. Both measure preference in a given decision context. Utility has the additional consideration of probabilities of various desired and undesired outcomes, as well as possibly other factors. Bouyssou and Vansnick (1988: 109) state, "That some idea of strength of preference interferes with risky choices is hardly disputable." They continue (110), "It seems many features do influence risky choice apart from strength of preference. Among them, we feel regret, ... disappointment, ... the existence of a specific utility (or disutility) of gambling, the misperception of probabilities ... and the avoidance of ruin are the most important ones." They state (109) "according to Bell [under certain conditions] that value is related to utility either by an affine or an exponential transformation." They also offer the observation that just because value and utility are both on an interval scale that does not mean that there exists a function that is a linear transformation from one to the other. (107)

Von Winterfeldt and Edwards (1986: 213) observe that as of that writing, little of the literature addressed the relationship between value functions and utility functions. They summarized the concept of mapping from the decision space into a preference space in a graph, adapted in Figure 3. An attribute is measured on some scale, transformed into value, and then

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the corresponding value is transformed into utility. Bypassing intermediate steps is also possible, as shown in Figure 4.

Objects → Natural or Constructed Scale → Value Scale ► Utility Scale

Locations of → Driving Distance → Ratings of Relative → Utilities of Values of
Apartments   (in miles) from   Value of Driving   Driving Distances
                 Office   Distances

L_i ⟶ $d(Li)$ ⟶ $v(d)$ ⟶ $u(v)$
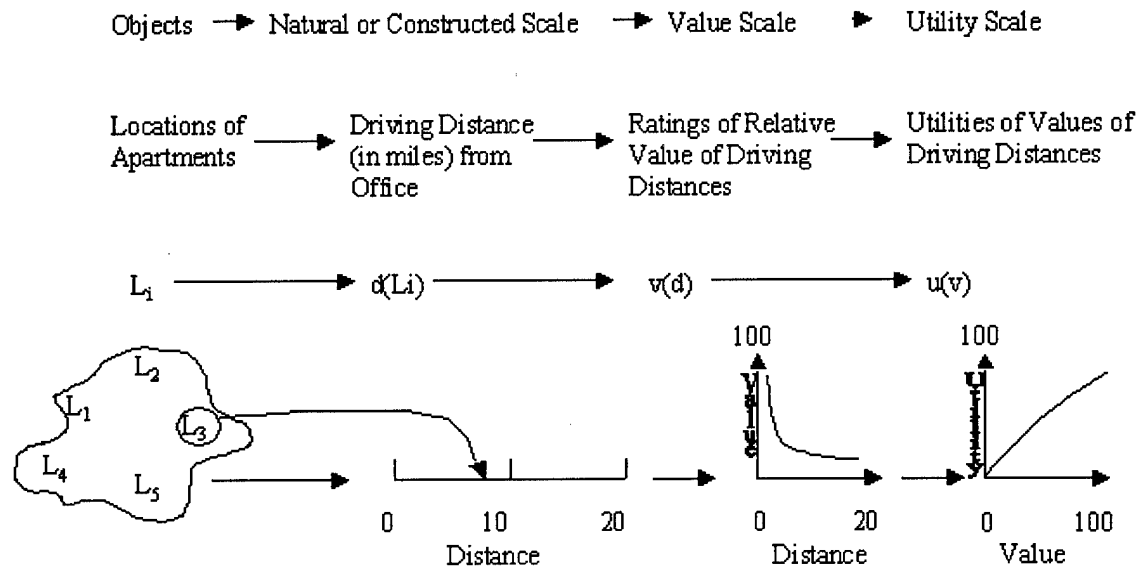
Figure 3. Mapping From Decision Space Into Preference Space (Von Winterfeldt and Edwards, 1986).

Full Decomposition

Objects → Natural or Constructed Scale → Value Scale → Utility Scale

No Value Scale (Keeney and Raiffa, 1976)

Objects → Natural or Constructed Scale ⟶ Utility Scale

No Natural or Constructed Scales (Bell and Raiffa, 1979)
Objects ⟶ Value Scale → Utility Scale

No Natural or Constructed Scale, Value Equals Utility (Edward, 1977)

Objects ⟶ Value Scale = Utility Scale

No Natural, Constructed or Value Scales (Raiffa, 1968)

Objects ⟶ Utility Scale

Figure 4. Decision Space To Utility Space Mapping, after von Winterfeldt And Edwards (1986).

Krzysztofowicz (1983b) builds on the work of Pratt (1964) and makes the argument that $u = w(v)$ for some transformation w. He developed various transforms for Pratt's *measure of local risk aversion*. Examining empirical data led him to the conclusion that value and utility functions are different measures, both in theory and as supported in use.

Keller (1985b) examined the relationship between value and utility functions in 29 subjects for one of four decision situations. She found that generally $v(x) \neq u(x)$ and that risk attitudes varied widely between subjects and between attributes for a given subject. She also observes that her findings are in disagreement of Krzysztofowicz (1983b), who found the exponential family to provide good results

2.4   Decision Analysis Methodologies

A number of textbooks are dedicated to the subject of decision analysis and offer methodological approaches. A selection of the more influential, in order of increasing age, is Clemen and Reilly (2001), Kirkwood (1997b), Keeney and Raiffa (1993), Kleindorfer, Kunreather, and Schoemaker (1993), Watson and Buede (1987), and von Winterfeldt and Edwards (1986). Additionally, the work by Keeney (1992a), while not a textbook, has been very influential in decision analysis.

2.4.1 SMARTS.

A number of methods employing the underlying tenets of value and utility for decision analysis have been developed. Within the decision analysis community, the SMARTS (Simple Multiattribute Rating Technique using Swings) technique is well known. Originally developed by Edwards (1977) it has undergone minor modifications over time. It is summarized in Edwards and Barron (1994) along with some simplifying modifications not discussed here. The steps of SMARTS, in the author's terminology, are:

1. Purpose and Decision Makers. Identify the purpose of the decision and those who will make it.
2. Value Tree (Hierarchy). Elicit a structured view of the objectives for the decision.
3. Objects of Evaluation. Define the alternatives with their attribute scoring scales and domain limits.

4. Objects-by-Attributes Matrix. Create a matrix comparing alternatives and scores.

5. Dominated Options. Remove from consideration all dominated alternatives.

6. Single-Dimensional Utilities. Elicit, and then replace the scores in the matrix with utility levels.

7. Rank Order Attributes. Order in declining importance.

8. Quantify Attribute Weights. Quantify in terms the most important attribute.

9. Decide. Analyze the decision, employing an overall utility model. The most common is the additive model.

10. Determine the utility of each alternative as given by

$$U_h = \sum_{i=1}^{I} w_i E\left[u_i\left(x_{hi}\right)\right] \tag{2}$$

where h ($h = 1, 2, \cdots, H$) is an index of alternatives, i ($i = 1, 2, \cdots, I$) is an index of attributes, w_i are the weights, x_{hi} is the ith evaluation measure under the hth alternative, and $u_i\left(x_{hi}\right)$ are the corresponding single dimensional utility levels.

For our purposes, without loss of generality, the preference functions will range in the unit interval and all attribute weights sum to one. That is, $u(x_i), v(x_i) \in [0,1] = \{r \in \mathbb{R} : 0 \le r \le 1\}$ and $\sum_{k=1}^{K} w_k = 1$ with $0 \le w_k \le 1$ for all k. Further, in this chapter, increasing levels of x are preferred. Define x_0 as the least preferred level of x and x_* as the most preferred. Then we have $u(x_0) = v(x_0) = 0$ and $u(x_*) = v(x_*) = 1$. These conventions are well-known in the decision analysis literature.

2.4.2 Utility Elicitation.

There are several approaches for assessing the single dimensional utility functions. The Lock-Step Procedure selects points in the attribute domain at some fixed interval (Kirkwood, 1997: 233). At each point, indifference curves are constructed permitting a utility function to be established. A more efficient, and widely used, approach is the Midvalue Splitting Technique (Kirkwood, 1997b: 233 – 235, 235 – 237). The domain is bisected, and the utility elicited at that

point. The two half-domains are similarly bisected, and those points elicited. This procedure may be extended until the desired resolution is achieved.

The elicitation of the utility at these points is frequently done employing one of two techniques, the certainty equivalent method or the probability equivalent method. In both methods, the decision maker is presented a decision involving a choice between uncertain alternative A and a certain alternative B. Alternative A has two possible outcomes. One occurs with probability p and yields outcome x_*, the most preferred level of x. The other occurs with probability $(1-p)$ and produces x_0, the least preferred level of x. Alternative B provides a sure payoff of x_B, where $x_0 < x_B < x_*$. This is shown in Figure 5.
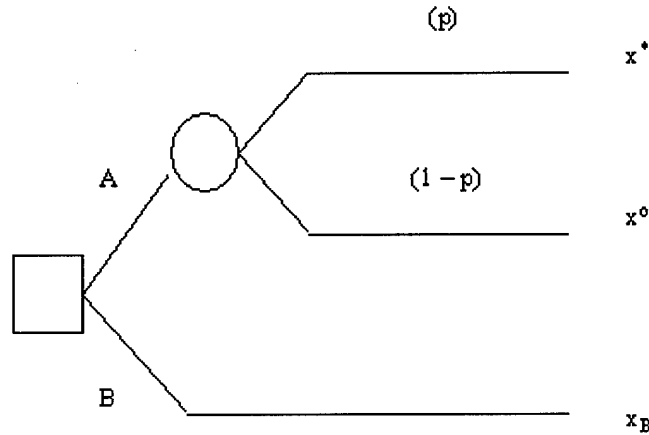
Figure 5. Decision Diagram. Squares represent decisions, circles uncertain events, parentheticals probabilities, and outcomes expressed as the variable x.

In the certainty equivalent method, the probability p is equal to 0.50, so Alternative A is a 50-50 bet between the high and low x values under consideration. The decision maker is presented various values of x_B until she is ambivalent in choosing between the two alternatives. By the expected utility axioms, the utilities of both alternatives must be equal at ambivalence. Further, the utility of x_* is equal to one, and the utility of x_0 is equal to zero, by definition. So

$$u(x_B) = (1-p)u(x_0) + (p)u(x_*) \tag{3}$$

$$u(x_B) = (.5)u(x_0) + (.5)u(x_*) \tag{4}$$

$$u(x_B) = (.5)(0) + (.5)(1) = .5 \tag{5}$$

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The utility may next be determined for the midpoints of the subregions $[x_0, x_B]$ and $[x_B, x_*]$. In the probability equivalent method, x_B is fixed at some value and the probability p is varied until the decision maker is ambivalent between the alternatives. This provides

$$u(x_B) = (1 - p)u(x_0) + (p)u(x_*) \tag{6}$$

$$u(x_B) = (1 - p)(0) + (p)(1) = p \tag{7}$$

Since p is known, the utility of x_B is known. The utilities of subregions are then determined in similar fashion as in the certainty equivalent method.

One the utility has been elicited at discrete points the utility function is established. A typical assumption, that of constant risk aversion permits the fitting of an exponential curve. In practice, often only a single midvalue point is used, and the curve fitted. (Kirkwood, 1997b: 235)

2.4.3 Value Elicitation.

Comparing the increase in preference for pairs of attribute levels permits value function elicitation. Often this is done my determining a midvalue level. If a decision maker is indifferent, for a given price paid, to go from x_0 to x_m as from x_m to x_*, then x_m has value equal to 0.5. This from

$$(x_0, x_m) \sim (x_m, x_*) \Rightarrow \left[v(x_m) - v(x_0) \right] = \left[v(x_*) - v(x_m) \right] \tag{8}$$

$$v(x_m) - 0 = 1 - v(x_m) \tag{9}$$

$$2v(x_m) = 1 \tag{10}$$

$$v(x_m) = \frac{1}{2} \tag{11}$$

Similar comparisons over the resultant subregions permit additional value point estimates.

2.4.4 Weight Elicitation.

Essentially, this is typically performed by starting with all attributes at their lowest levels, then "swinging" one attribute at a time from its lowest to highest levels. Take the smallest reported increase in worth, and then express the other increase in terms of the smallest. Normalize the weights so that they sum to one. (Kirkwood, 1997b: 68 – 70) Clemen (1996: 546

– 552) also provides a description of this technique as well as pricing out and lottery weight techniques.

2.5 Sensitivity Analysis.

Sensitivity analysis has a role in all decision making, not just within decision analysis. "In solving a problem, a sensitivity analysis should be made in order to determine the variables to which the outcome is most sensitive" (Operations Analysis Study Group, 1977: 12). A typical decision analysis effort involves first constructing a deterministic model of the decision opportunity. Then the model is examined to see in where uncertainty causes important effects. Those variables where uncertainty has a significant impact are then modeled as stochastic while the remaining variables are handled in a deterministic manner. The determination of which variables should become stochastic members of the model is called *sensitivity analysis* and is and is ascribed to Howard and sometimes referred to as the "Stanford approach" (Reilly, T., 2000: 551). "The motivation behind sensitivity analysis is to reduce the assessment burden" (Reilly, T., 2000: 556). Eschenbach (1992: 41) defines sensitivity analysis as "examining the impact of reasonable changes in base-case assumptions." Skinner (1999: 358) links the employment of sensitivity analysis specifically to uncertain aspects of a decision problem, rather than assumptions. Eschenbach provides that the concern is for the sensitivity of the response variable to differing levels of independent variables. He lists several considerations for sensitivity analyses: (1) establish reasonable bounds for independent variables, (2) establish the "unit impact" of these changes, (3) establish the maximum impact of the independent variables, and (4) establish the change required in each independent variable to effect alternative choices.

Klimack (2002) summarizes several approaches to sensitivity analysis in DA. The traditional approach has been termed the "Stanford Approach" and involves perturbing each variable of a deterministic DA model, usually singly, and observing the results. Sensitivity analysis may be accomplished through a technique referred to as Rank Order Stability Analysis (ROSA) and attributed to Einarson, Arikian, and Doyle (1995) by Jolsh and Armstrong (2000: 537 – 538). This approach is basically that of a standard sensitivity analysis. Instead of varying a parameter through a range of values of interest and observing the response of the objective function and noting the change in optimum alternative, ROSA merely notes the range for the variable that retains the optimum alternative. Dependent sensitivity analysis (DSA), for

sensitivity analysis where the singular value decomposition of a correlation matrix is used to combine the independent variables into approximately uncorrelated synthetic variables Reilly (2000). Bauer, Parnell and Meyers (1999: 162 – 180), demonstrated the efficacy of response surface methodology (RSM) for decision analysis sensitivity analysis. Response surface methodology is "a collection of statistical and mathematical techniques useful for developing, improving, and optimizing processes" (Myers and Montgomery, 1995: 1).

2.5.1 Response Surface Methodology.

Response surface methodology provides the ability to model some system that behaves according to $y = g(x_1, x_2, \ldots, x_n) + \varepsilon$, where g is unknown, with $y = f(x_1, x_2, \ldots, x_n) + \varepsilon$, where f is determined with empirical data. Generally f is either a first or second-degree polynomial, and forms the response surface. Figure 6 illustrates how a system may be viewed as transforming inputs ξ_i into some response y or $g(\xi_1, \xi_2, \ldots, \xi_n) = y$. Response surface methodology replaces g with a low degree polynomial f. Response surface methodology is frequently used as a substitute for g or to gain insight into the important aspects of g. Response surface methodology is sometimes referred to as regression analysis or meta-modeling. (Bauer, 2001: 71)
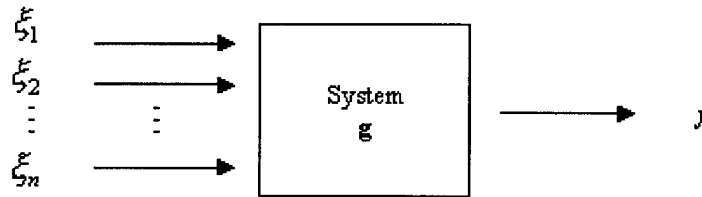
Figure 6. Representation of a System Modeled by RSM.

Response surface methodology may be thought of a polynomial surface over a region that approximates the surface of g. Each alternative in a decision would generally have a differing surface g. The independent variables, ξ_i, have upper and lower bounds L_i and U_i respectively, when considered across all alternatives. Generally in RSM the independent variables are coded

$$x_i = 2\left(\frac{\xi_i - \xi_{i0}}{U_i - L_i}\right) \text{ for } i = 1, 2, \ldots, n \text{ where } \xi_{i0} \text{ is the midpoint between } L_i \text{ and } U_i. \text{ This maps } L_i$$

and U_i to -1 and $+1$, respectively, offering the advantage of working with unitless variables and facilitating experimental design. (Bauer, Parnell, Meyers, 1999: 164)

Response surface methodology produces a metamodel which models the response of the decision analysis model as the input is varied. The response is the multiattribute preference function, and the input the decision parameters. The hypothesized model describes the response

$$y = \beta_0 + \sum_{i=1}^{n} \beta_i x_i + \sum_{i<j} \beta_{ij} x_i x_j + \sum_{i=1}^{n} \beta_{ii} x_i^2, \quad \varepsilon \sim N(0, \sigma^2) \tag{12}$$

where the β are unknown coefficients. This approach is analogous to a Taylor's series expansion of degree 2 for the true functional relationship. These coefficients provide insight as to contribution of each variable, as well as the strength of variable interactions. Equation (12) may be simplified by omitting the x_i^2 term and the $x_i x_j$ interaction term may also to neglected. As simplified, the expression is easier to solve and so is often used as a screening device early in the RSM process. Employing the complete Equation (12) near the subregion of concern provides a better fit of the approximated surface. The least squares estimated coefficients of the hypothesized model

$$\hat{y} = b_0 + \sum_{i=1}^{n} b_i x_i + \sum_{i<j} b_{ij} x_i x_j + \sum_{i=1}^{n} b_{ii} x_i^2 \tag{13}$$

is, in vector notation,

$$\mathbf{b} = (\mathbf{X'X})^{-1} \mathbf{X'y} \tag{14}$$

The fitted surface is provided by

$$\hat{\mathbf{y}} = \mathbf{Xb} \tag{15}$$

and residual error is

$$\mathbf{e} = \mathbf{y} - \hat{\mathbf{y}} \tag{16}$$

The coefficient provides a ready indicator of the contribution for the associated variable and statistical tests provide a measure of significance. (Myers and Montgomery, 1995: 16 – 21)

Bauer, Parnell, and Meyers present Figure 7, a graphical representation of iterative application of RSM to a DA problem (1999: 165 – 166). To reduce the analytic burden, variables that are believed to behave similarly may be grouped and a RSM screening conducted. Those groups that have significant effects are then screened as individual factors. Significant

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variables are then modeled with a first order equation and the fit is tested. If the fit is insufficient, a second order model is employed.

Group 1 ⎰
Group 2 ⎱   GROUP
Group 3 —   SCREENING ———— y

Group 1 ⎰   FACTOR
              SCREENING ———— y
Group 3 —

FIRST ORDER
LINEAR ———— y
MODELING

LACK OF FIT

SECOND ORDER
LINEAR ———— y
MODELING
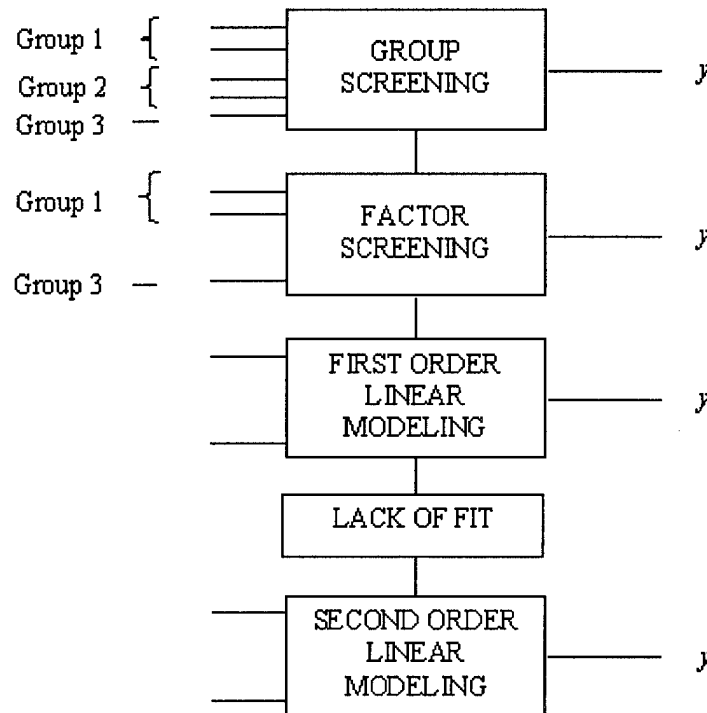
Figure 7. RSM Paradigm (After Bauer, Parnell, and Meyers, [1999: 166]).

2.5.2 Group Screening.

As Bauer, Parnell, and Meyers (1999) employ group screening, it deserves additional discussion beyond a review of RSM. Meyers and Montgomery (1995) fail to mention group screening in their RSM text. A screening design in experimentation is a selection of experimental inputs that permit establishment of which factors are significant while minimizing the number of experimental iterations (usually referred to as runs in modeling) and so reducing associated costs. Large numbers of factors force a prohibitively large number of runs or, when the number of runs is constrained, causes factors to be confounded and effects masked. Madu and Kuei (1992: 96) caution that screening large numbers of variables is to be avoided. Group screening reduces the number of factors considered by consolidating them into groups. These

groups are then manipulated simultaneously so the experimental design has been reduced from the total number of factors to the number of groups.

In the procedure, stated simply, variables are first aggregated into groups. Various authors have developed recommended differing grouping assumptions and strategies. Then an acceptable experimental design based on the groups is selected. When the runs are complete, all variables contained in the groups found not to be significant are assumed to be non-significant. Significant groups are decomposed and new groups formed, if required, and the process repeated. Their efforts are shown graphically in Figure 7.

While standard decision analysis sensitivity analysis identifies which variables, without interaction, cause the solution to be sensitive, and DSA addresses interactions employing synthetic variables, RSM provides a quantitative estimate of the impact of each variable and interactions. Sensitivity analysis in the decision analysis literature excludes discussion of preference functions. The literature does not address the issue, although it does provide evidence that the issue is of import. Ghosh and Ray (1997) found that decision maker's choices are both a function of risk attitudes and ambiguity tolerance. Additionally, their results suggest that the decision maker often interprets ambiguity in a decision situation as risk. This indicates that the preference function likely involves some inherent uncertainty.

Besides the uncertainty present within the decision maker, elicitation sessions are vulnerable to human perception frailties. An example of this includes framing, where the same situation presented in differing manners elicits consistently differing preferences. Anchoring is the phenomenon in which an individual presented with some estimate of a parameter of interest will often provide biased elicitation, assessing higher probabilities against the estimate while significantly underestimating probabilities of severe events. Further, it is well documented that individuals will demonstrate more confidence in objective probabilities than subjective probabilities without discernable justification. See Kirkwood (1997b: 110 – 127; 299 – 320) for discussion of these issues.

3 Hybrid Value-Utility Decision Analysis Models

3.1 General

As discussed previously, decision analysis models may be utility-function-based, or value-function-based. Klimack (2002b) showed that the two models differ significantly for military decision-making. The utility-based model is appropriate when outcomes are not certain, as is typical for difficult decisions. As utility functions incorporate the risk attitudes of an individual, they are specific to an individual. Therefore construction of utility models requires a significant amount of the decision maker's time. For large decision problems, this personal commitment on the part of the decision maker is onerous and is a barrier to the use of multiple objective decision analysis. Value-based models, appropriate under conditions of certainty, may be constructed through consultation with appropriate experts (e.g., a military staff) rather than the decision maker. Consultation with subject matter experts is generally required in any complex problem, regardless of methodology applied.

If a utility-based model may be approximated by modification of a value-based model, reduction of the time demands on the decision maker may be realized. For large, important decisions, these time demands are significant. Reducing the time required of the decision maker improves the acceptance of the procedure and improves the probability that decision analysis will be employed. This chapter will first review pertinent background information. Then an algorithm will be presented for construction of the hybrid model.

3.2 Background

Decision analysis models consist of a set of evaluation criteria for which preference functions are used to assess the decision maker's strength of preference for various levels of the criteria, and often also attitudes regarding the uncertainty of the decision situation. In a typical value-focused approach (Keeney, 1992), the evaluation criteria are determined through decomposition of the decision maker's strategic priorities until measurable criteria, the evaluation measures, are obtained. This inverted dendritic structure, the value hierarchy, is

shown in the upper portion of Figure 8. Each evaluation measure is assessed using a single dimensional preference function. Under conditions of certainty, the preference function is referred to as a value function and measures the decision maker's strength of preference. Under uncertainty, the preference function is referred to as a utility function and measures both the decision maker's strength of preference and her attitude toward the risk inherent in the uncertain decision context. The preference functions are contained in the second from bottom row of Figure 8. A multiattribute (multidimensional) function employing relative weights (relative importance) of the various evaluation measures combines the single dimensional preference functions and completes the model, permitting evaluation of alternatives. Figure 8 represents a common view of decision analysis models, which assumes perfect information. (In fairness, it must be observed that sensitivity analysis is often used to examine the assumption of perfect information.)
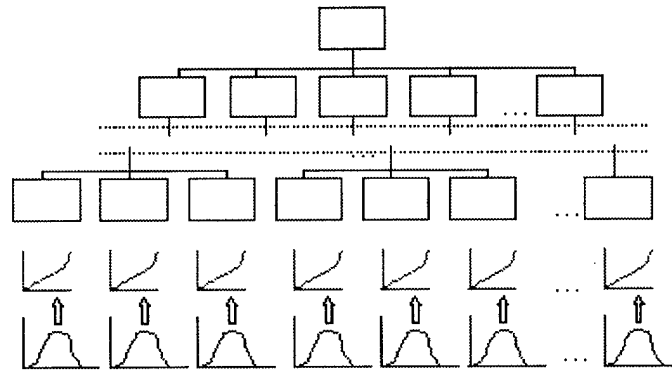
Figure 8. Common Decision Analysis Model Structure. The bottom tier represents the evaluation measures and the next tier the preference functions. The hierarchy represents the decomposed values of the decision maker.

A multiattribute function for a utility model is often written in the additive form. This is

$$U_j = \sum_{i=1}^{I} E\left[w_i u_i \left(x_i \right) \right] \tag{17}$$

where U_j is the utility of the jth alternative, w_i is the weight (relative importance) of the ith evaluation measure, and $u_i \left(x_i \right)$ is the single dimensional utility function for the ith evaluation measure, x_i is the random variable for the ith evaluation measure, and I is the number of evaluation measures.

Equation (17) may be rewritten in terms of the utility probability density function, or

$$E[U_j] = E\left[\sum_{i=1}^{I} w_i f_i''(u_i(x_i), \hat{\alpha}_{ij}')\right] = \sum_{i=1}^{I} w_i E\left[f_i''(u_i(x_i), \hat{\alpha}_{ij}')\right] \qquad (18)$$

where f_i'' is the probability density function of the ith utility function, and $\hat{\alpha}_{ij}'$ is a vector of parameters of the estimated evaluation measure distributions. The DA problem may be stated as maximizing $E[U_j]$ through the selection of the jth alternative. More formally, we take the maximum

$$\text{Max } E[U_j] \qquad (19)$$

such that

$$U_j \in \{U_1, U_2, \ldots, U_J\} \qquad (20)$$

$$E[U_j] = E\left[\sum_{i=1}^{I} w_i f_i''(u_i(x_i), \hat{\alpha}_{ij}'')\right] \qquad (21)$$

$$i = 1, \ldots, I \qquad (22)$$

$$j = 1, \ldots, J \qquad (23)$$

For a value-based model, the additive multiattribute function is

$$V_j = \sum_{i=1}^{I} w_i v_i(x_i) \qquad (24)$$

where V_j is the value of the jth alternative and $v_i(x_i)$ is the single dimensional value function for the ith evaluation measure. Equation (24) contains no expectation operator, as it is intended for us under conditions of certainty. However, using value functions under uncertainty produces a value pdf that behaves similarly to the utility pdf. Equation (24) then becomes

$$V_j = \sum_{i=1}^{I} E[w_i v_i(x_i)] \qquad (25)$$

In analogous fashion, an objective value function may be constructed

$$\text{Max } E[V_j] \qquad (26)$$

such that

$$V_j \in \{V_1, V_2, \ldots, V_J\} \qquad (27)$$

$$E[V_j] = E\left[\sum_{i=1}^{I} w_i f_i'(v_i(x_i), \hat{\alpha}_{ij}')\right] \qquad (28)$$

$$i = 1, \ldots, I \qquad (29)$$

$$j = 1, \ldots, J \qquad (30)$$

where f_i' is the probability density function of the ith value function, and $\hat{\alpha}_{ij}'$ is a vector of parameters of the estimated evaluation measure distributions. The DA problem may be stated as maximizing $E[V_j]$ through the selection of the jth alternative.

In uncertain decision contexts Equation (18) is appropriate but for large problems of great import the $u_i(x_i)$ are difficult to elicit from the decision maker because of the time involved. As risk attitudes vary within an organization, it is not acceptable to use a surrogate for the decision maker to elicit utility functions. Arguably, an organization should have a consensus for values in a decision situation, so parameters for Equation (24) are may be elicited from subject matter experts. Modifying Equation (24) provides an approach to estimate Equation (17) to within some specified accuracy. The advantage of this approach is the reduction of the elicitation burden on the decision maker. This onerous burden sometimes either precludes employing a utility approach or causes the selection of non-decision analysis techniques, and has been identified as a limitation of this methodology.

The proposed procedure is depicted graphically in Figure 9. Starting with the value model, a hybrid model is created by replacing certain single dimension value functions with the corresponding utility functions. The resultant hybrid model is then used as would the utility model to analyze the decision situation.

Typically, the initial step of building the value model is done through interview of the decision maker. As we wish to minimize the time required of this individual, we will construct the value model using appropriate subject matter experts. The decision maker then should validate the value model. As we are concerned with complex decision situations, often we would expect the number of evaluation measures to be large. Group screening is a method that efficiently examines large numbers of variables. We anticipate that group screening will be employed as part of the step determining significance of the evaluation measures. Further development of the methodology outlined in Figure 9 is provided in the next chapter.
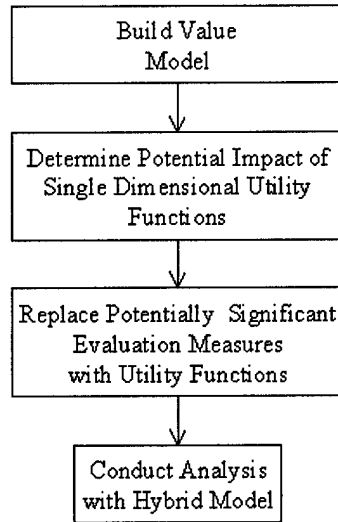
Figure 9. Basic Hybrid Value-Utility Model Approach.

4 A Hybrid Value-Utility Algorithm

4.1 Algorithm Development.

A method of approximating Equation (17) is desired. Beginning with Equation (24), one evaluation measure's value function, $v_k(x_k)$, is replaced with the appropriate utility function, $u_k(x_k)$. The model becomes

$$E\left[\hat{U}_j^{(1)}\right] = E\left[\sum_{i=1,i\neq k}^{I} w_i f_i'\left(v_i(x_i),\hat{\alpha}_{ij}'\right) + w_k f_k''\left(u_k(x_k),\hat{\alpha}_{ij}''\right)\right] \tag{31}$$

where the kth evaluation measure value function has been replaced with the corresponding utility function and $\hat{U}_j^{(1)}$ is an estimate of U_j after one substitution.

Equation (31) then becomes, after m substitutions,

$$E\left[\hat{U}_j^{(m)}\right] = E\left[\sum_{i=1}^{I}\left[(1-K_i)w_i f_i'\left(v_i(x_i),\hat{\alpha}_{ij}'\right) + K_i w_i f_i''\left(u_i(x_i),\hat{\alpha}_{ij}''\right)\right]\right] \tag{32}$$

where K_i is an indicator variable corresponding to the substitutions defined for the ith evaluation measure as

$$K_i = \begin{cases} 0 & \text{when } v_i(x_i) \text{ remains,} \\ 1 & \text{when } u_i(x_i) \text{ substituted} \end{cases} \tag{33}$$

This substitution protocol continues in an iterative fashion until the estimated utility, $\hat{U}_j^{(m)}$ is sufficiently close to U_j. This may be stated as $\left|E\left[\hat{U}_j^{(m)}\right] - E\left[U_j\right]\right| \leq \delta$, where δ is the level of accuracy desired. Then $E\left[\hat{U}_j^{(m)}\right] \approx E\left[U_j\right]$. Equation (32) is a hybrid value-utility model that estimates the true utility model behavior.

4.2 Substitution order

Two additional considerations must be addressed in order to develop an algorithm. These are the order of the substitutions of single dimensional utility for single dimensional value

functions, and when the substitutions may be halted. In general, the substitutions should be made in order of decreasing impact upon the hybrid model, equation (32). The coefficients of the regression model produced by RSM indicate the potential contribution of each model variable and permit the prioritization of substitution of the evaluation measures. The substitutions may be stopped when sufficient accuracy is achieved, as discussed below.

Frequently, utility functions are approximated employing parametric functions fitted to the data (Kirkwood, 1997: 65). When the exponential function,

$$\hat{u}_i\left(v_i\left(x_i\right)\right) = \frac{1 - e^{-v_i(x_i)/\rho_i}}{1 - e^{-1/\rho_i}} \tag{34}$$

is used to approximate a utility function the exponential constant, ρ, may be modified to examine an envelope of utility curves. Setting $\rho_i = \infty$ for $\hat{u}_i\left(v\left(x_i\right)\right)$ establishes that there are no risk attitudes and therefore $\hat{u}_i\left(v\left(x_i\right)\right) = v_i\left(x_i\right)$. Varying ρ_i permits employment of response surface methodology to determine whether each $\hat{u}_i\left(x_i\right)$ potentially exerts a significant effect on the hybrid model. Kirkwood (1997b: 66) indicates that the absolute value of ρ is typically greater than one-tenth of the evaluation measure domain, or $\rho_i \geq \left|\dfrac{\max x_i - \min x_i}{10}\right|$. The region bounded this relationship is illustrated in Figure 10. Accepting the limits proposed by Kirkwood means that if we examine the two $\hat{u}_i\left(x_i\right)$ for $\rho = \pm 0.1$, and they do not significantly affect the hybrid model, then the preference function for the i th evaluation measure may remain a value function. We do not expect the utility function for that evaluation measure to significantly affect the hybrid model.

4.3 Stopping Criteria

Replacing all potentially significant evaluation measures will ensure that the hybrid model adequately represents the utility model. However, it is desired to halt the iterative substitutions when $\left|E\left[\hat{U}_j^{(m)}\right] - E\left[U_j\right]\right| \leq \delta$, which may occur before all the evaluation measures

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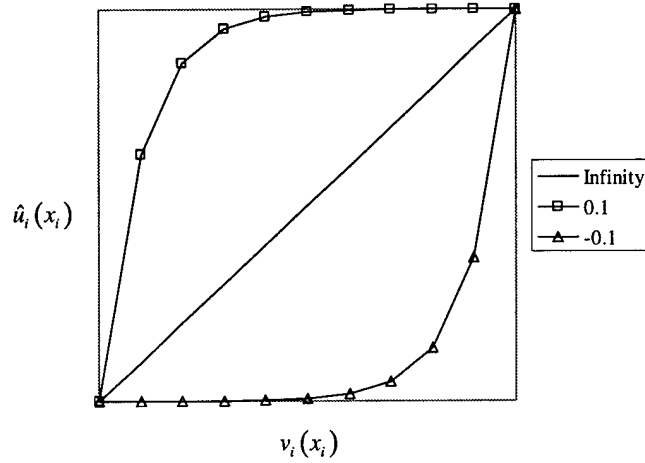
Figure 10. Exponential Utility Function Limits. The straight line, where $u_i(x_i) = v_i(x_i)$, occurs when $\rho = \infty$. The other curves show $u_i(x_i)$ when $\rho = \pm 0.1$.

have been converted. We may write the problem as a mathematical programming problem. The optimization model is:

$$\min \ m = \sum_{i=1}^{I} K_i \tag{35}$$

subject to

$$\left| E\left[\hat{U}_j^{(m)}\right] - E\left[U_j\right] \right| \leq \delta \ \forall j \tag{36}$$

$$\delta \in [0,1] \tag{37}$$

However, $E\left[U_j\right]$ is unknown, so it is not directly possible to determine when $E\left[\hat{U}_j^{(m)}\right]$ is within δ of $E\left[U_j\right]$. When there is consistent risk aversion (risk affinity) then the set of all possible $E\left[\hat{U}_j^{(m)}\right]$, S_j, is bounded below (above) by V_j and above (below) by $E\left[U_j\right]$. See Figure 11 for examples of behavior of preference function under various risk attitudes. When risk aversion is present for the ith evaluation measure, $u_i(v_i(x_i)) \geq v_i(x_i)$. The reverse is found under risk seeking behavior. Mixed risk attitudes preclude such general conclusions
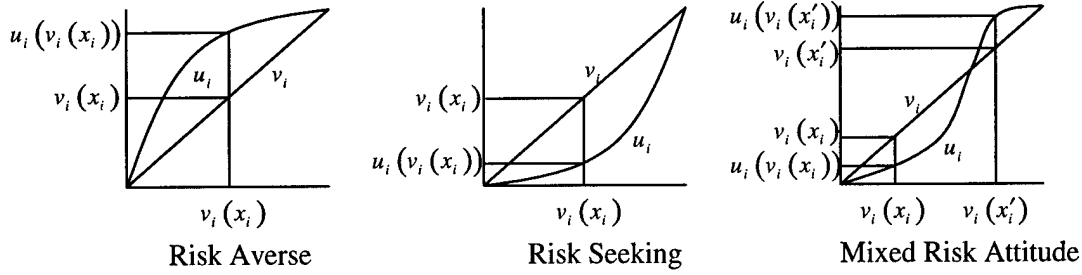
Figure 11. Risk Attitudinal Effects on the Relationship of Value and Utility Functions.

A useful measure is the substitution distance, the difference between successive substitutions. Given a series of substitutions to create hybrid value-utility models for the jth alternative, $S_j = \left\{ V_j, \hat{U}_j^{(1)}, \hat{U}_j^{(2)}, \ldots, \hat{U}_j^{(m)} \right\}$, the distance between successive substitutions is given by $\lambda_j^{(m)}$, defined as

$$\lambda_j^{(m)} = \left| E\left[\hat{U}_j^{(m)} \right] - E\left[\hat{U}_j^{(m-1)} \right] \right| \tag{38}$$

where m indicates the number of substitutions, and $E\left[\hat{U}_j^{(0)} \right] = E\left[V_j \right]$.

When the set S_j is ordered with decreasing intervals between iterative estimates, $\lambda_j^{(m)} \leq \lambda_j^{(m-1)}, \forall m \in \{2,3,\ldots,I\}$, then S_j is a partially ordered set. Under this condition, when $(I-m)\lambda_j^{(m)} \leq \delta$, then $\left| E\left[\hat{U}_j^{(m)} \right] - E\left[U_j \right] \right| \leq \delta$. When $(I-m)\lambda_j^{(m)} \leq \delta$ is encountered the substitutions of single dimensional utility functions for value functions is halted and the current $E\left[U_j^{(m)} \right]$ accepted as accurately estimating $E\left[U_j \right]$.

There are a number of potential alternatives of interest, not just alternative j. Define the vectors $\mathbf{U}$ and $\hat{\mathbf{U}}^{(m)}$

$$\mathbf{U} = \begin{bmatrix} U_1 \\ \vdots \\ U_j \\ \vdots \\ U_J \end{bmatrix} \text{ and } \hat{\mathbf{U}}^{(m)} = \begin{bmatrix} \hat{U}_1^{(m)} \\ \vdots \\ U_j^{(m)} \\ \vdots \\ U_j^{(m)} \end{bmatrix} \tag{39}$$

Clearly it is not necessary to continue iterations, and so to increment m, when $\left| E\left[\hat{\mathbf{U}}_j^{(m)} \right] - E\left[\mathbf{U}_j \right] \right| \leq \delta \ \forall j$. What is of interest is, naturally, the alternative with the greatest utility. Therefore we are satisfied when the alternative with the greatest utility is alone within the accepted tolerance. Define $\hat{U}_*^{(m)} = \max\left\{ \hat{U}_1^{(m)}, \hat{U}_2^{(m)}, \cdots, \hat{U}_J^{(m)} \right\}$ where m is the most recent iteration. Define $\lambda_*^{(m)} = \left| E\left[\hat{U}_*^{(m)} \right] - E\left[\hat{U}_*^{(m-1)} \right] \right|$ where $*$ represents the evaluation measure with the greatest utility in the last, mth, iteration. The mathematical programming formulation now becomes:

$$\min \ m = \sum_{i=1}^{I} K_i \tag{40}$$

subject to

$$(I - m)\lambda_*^{(m)} \leq \delta \tag{41}$$

$$\delta \in [0,1] \tag{42}$$

When S_j is partially ordered, m is minimized when equation (41) is first found to hold. Response surface methodology provides the sufficient conditions for partial ordering of the substitution distances. Stop the algorithm when $(I - m)\lambda_*^{(m)} \leq \delta$ and accept $E\left[U_*^{(m)} \right]$ as accurately estimating $E[U_*]$ and the alternative corresponding to $U_*^{(m)}$ as being the optimal choice.

4.4 Group Screening

As stated previously, the class of decision analysis problems considered on which these techniques are likely to be beneficial are those with a large number of evaluation measures. For such problems, group screening is likely to be required. The group screening employment is as traditionally employed as presented in Chapter 2. Once the screening is completed, the information must be re-aggregated, which is a new contribution.

Employing groups screening permits examination of a large number of variables through decomposing the independent variables into subsets. Once the groups screening is completed, it

is desired to reassemble the variables into a single set, rank ordered by the degree of affect each has on the model. The utility model is

$$\hat{U} = \sum_{i=1}^{n} w_i \hat{u}_i \left(v_i \left(x_i \right) \right) \tag{43}$$

where, with the assumption of the exponential single dimensional model, the ith evaluation measure is determined by

$$\hat{u}_i \left(v_i \left(x_i \right) \right) = \frac{1 - e^{-v_i(x_i)/\rho_i}}{1 - e^{-1/\rho_i}} \tag{44}$$

When the kth element of (43) is examined separately, the model becomes

$$\hat{U} = \sum_{i=1}^{k-1} w_i \hat{u}_i \left(v_i \left(x_i \right) \right) + w_k \hat{u}_k \left(v_k \left(x_k \right) \right) + \sum_{i=k+1}^{n} c_i x_i \tag{45}$$

As equation (43) may be represented by a fit of a linear model based on the exponential constant

$$\hat{U} = c_0 + \sum_{i=1}^{n} c_i \rho_i \tag{46}$$

the kth element of (45) may be decomposed from (46) and modeled by a least squares model

$$u_k = c_0 + \sum_{i=1}^{k-1} c_i \rho_i + c_k \rho_k + \sum_{i=k+1}^{n} c_i \rho_i \tag{47}$$

The coefficient c_k provides a measure of importance of ρ_k relative to the other ρ_i.

If the kth element has been selected to be treated as a group of variables for screening, then it becomes necessary to decompose the kth group when it is determined to be significant. Because the kth group may also be modeled by

$$u_k = c_{k0} + \sum_{i=1}^{n} c_{ki} \rho_{ki} \tag{48}$$

the contribution to the overall model for the ρ_{km} th variable is $c_k c_{km}$. Similar combinations are made for all grouped variable coefficients to permit comparison of evaluation measure priorities. The relative importance of utility functions for the evaluation measures is provided by the corresponding $c_k c_{km} \cdots c_{k(r-1)} c_{kr}$ where r is the number of screening sets in which the kth evaluation measure has been grouped. Rank ordering the evaluation measures by relative importance provides the order for the substitutions from single dimensional value function to single dimensional utility functions when the hybrid model is created.

4.5 The Detailed Algorithm

Refining the basic concept provided in Figure 9 by minimizing the interactions required with the decision maker, incorporating group screening, and using response surface methodology to assess potential significance provides the algorithm presented in Figure 12.

The steps of this protocol are:

1.   Build Value Model.

2.   Establish the multiattribute functional form. Typically the simple additive form is used, but the algorithm is independent of the multiattribute functional form. A constraint on the algorithm is that the value model and the utility model must be of the same form.

3.   Elicit value hierarchy and evaluation measure weights, w_i, from subject matter experts. This avoids a significant time burden being placed upon the decision maker.

4.   Verify the value hierarchy and evaluation measure weights with decision maker. Clearly the model must be acceptable to the decision maker. Verification at this point provides efficient elicitation of the single dimensional value functions.

5.   Elicit $v_i(x_i) \forall i$ from subject matter experts.

6.   Determine potential impact of single dimensional utility functions.

   a.   Group $v_i(x_i)$. Grouping is done to permit group screening, which allows examination of large numbers of evaluation measures. The value hierarchy provides a ready framework for grouping of variables. An obvious approach is to group variables at the highest partitioning. Each group may be subsequently examined, employing group screening as required. An additional consideration is that if any interactions are believed to be present, the groupings should not separate those variables.

   b.   Develop and execute experimental design. Design of experiments permits the significance of the predictor variables to be assessed with the minimum number of experimental runs. A run in this sense, is setting the variables at some level and observing the expected utilities of the various alternatives. The variables of interest with respect to the hybrid model are the exponential

```
┌─────────────────────────┐        ┌─────────────────────────────────────────┐
│    Build Value Model    │        │   Establish Multiattribute Functional Form │
└─────────────────────────┘        └─────────────────────────────────────────┘
                                                      │
                                                      ▼
                                   ┌─────────────────────────────────────────┐
                                   │   Elicit Value Hierarchy and Evaluation   │
                                   │   Measure Weights From Subject Matter      │
                                   │                 Experts                    │
                                   └─────────────────────────────────────────┘
                                                      │
                                                      ▼
                                   ┌─────────────────────────────────────────┐
                                   │   Verify Value Hierarchy and Evaluation   │
                                   │   Measure Weights With Decision Maker      │
                                   └─────────────────────────────────────────┘
                                                      │
                                                      ▼
                                   ┌─────────────────────────────────────────┐
                                   │  Elicit Single Dimensional Value Functions │
                                   │        from Subject Matter Experts         │
                                   └─────────────────────────────────────────┘
                                                      │
                                                      ▼
┌─────────────────────────┐        ┌─────────────────────────────────────────┐
│ Determine Potential      │        │        Group Evaluation Measures          │
│ Impact of Single         │        └─────────────────────────────────────────┘
│ Dimensional Utility      │                         │
│ Functions                │                         ▼
└─────────────────────────┘        ┌─────────────────────────────────────────┐
                                   │      Design and Execute Experiment         │
                                   └─────────────────────────────────────────┘
                                                 │        ▲
                                                 ▼        │
                                   ┌─────────────────────────────────────────┐
                                   │      Employ RSM to Evaluate Potential      │
                                   │  Significance of Each Evaluation Measure   │
                                   └─────────────────────────────────────────┘
                                                      │
                                                      ▼
                                   ┌─────────────────────────────────────────┐
                                   │      Ungroup Variables and Determine       │
                                   │         Overall Relative Importance        │
                                   └─────────────────────────────────────────┘
                                                      │
                                                      ▼
┌─────────────────────────┐        ┌─────────────────────────────────────────┐
│ Replace Potentially      │        │     Replace Most Potentially Significant    │
│ Significant Evaluation   │        │   Single Dimensional Value Function with    │
│ Measure with Utility     │        │    the Corresponding Utility Function       │
└─────────────────────────┘        └─────────────────────────────────────────┘
                                                 │        ▲
                                                 ▼        │
                                   ┌─────────────────────────────────────────┐
                                   │     Determine if Hybrid Model is Adequate   │
                                   └─────────────────────────────────────────┘
                                                      │
                                                      ▼
┌─────────────────────────┐        ┌─────────────────────────────────────────┐
│ Conduct Analysis with    │        │  Select Best Alternative Based on Expected │
│ Hybrid Model             │        │                  Values                    │
└─────────────────────────┘        └─────────────────────────────────────────┘
                                                      │
                                                      ▼
                                   ┌─────────────────────────────────────────┐
                                   │        Perform Sensitivity Analysis        │
                                   └─────────────────────────────────────────┘
```
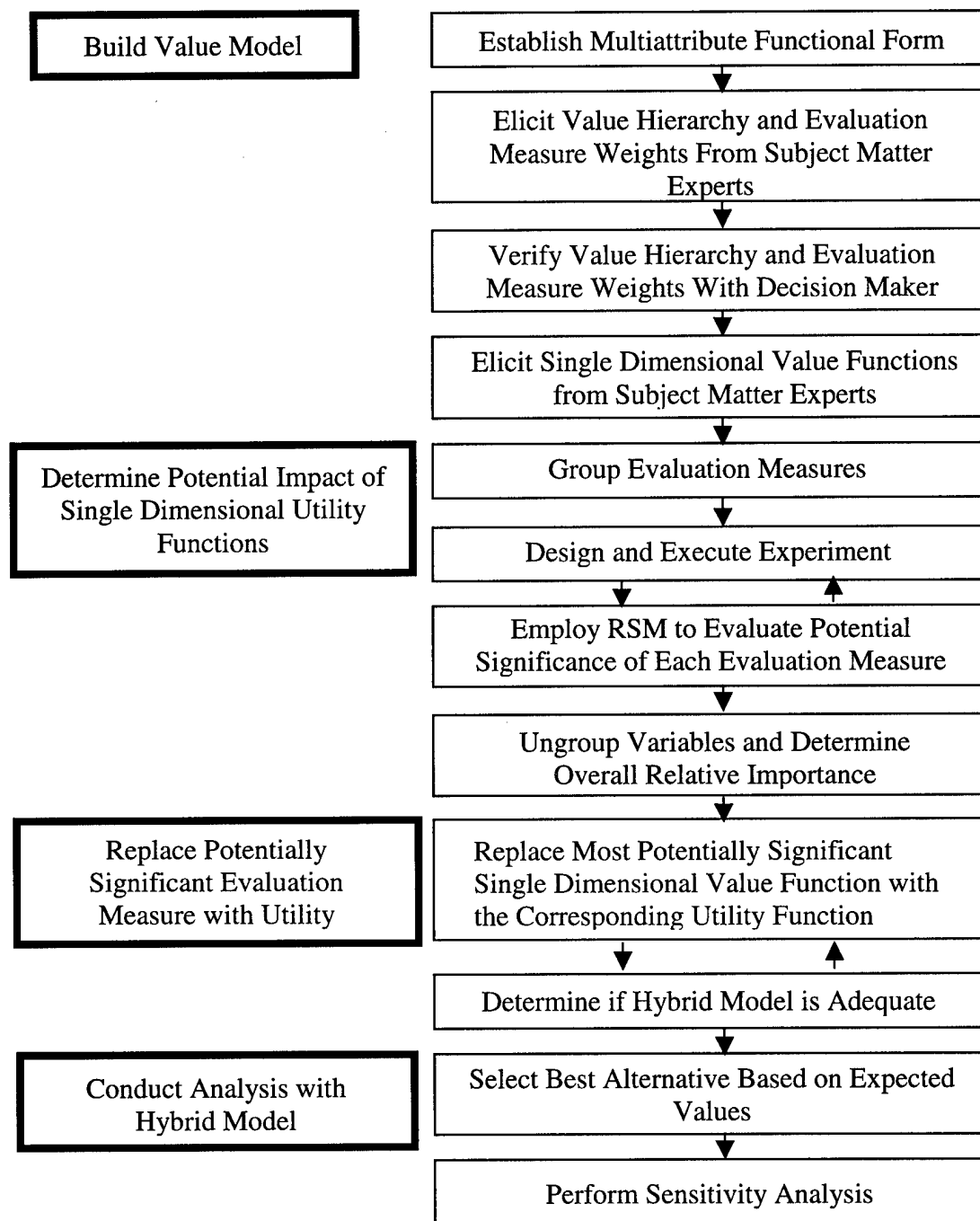
Figure 12. Steps to Create Hybrid Value-Utility Model.

constants of the $\hat{u}_i(v_i(x_i))$ functions. As the exponential constants assume

values of $\rho = \{-0.1, \infty, 0.1\}$, the fractional factorial with center point designs

are attractive. The experimental design is complicated by the consideration

of the set of alternatives. The alternative choice may be included as a

variable in the experimental design. If the set is large, the number of

available experimental designs is small. This step and the following step are

repeated for each grouping of variables.

c. Employ RSM. Examine potential contribution of $u_i(x_i)$ in the current

model in place of $v_i(x_i)$ by employing response surface methodology to

examine $\hat{u}_i(v_i(x_i))$. This analysis provides regression coefficients showing

the contribution of each variable as well as significance information. This

provides data that facilitates determining the evaluation measures that need to

be considered for conversion into utility, at some defined alpha level, as well

as straightforward prioritization information. This step is done for each

grouping of variables.

d. Ungroup $v_i(x_i)$. The RSM information from the previous step is

ungrouped to provide a common comparison for all evaluation measures.

Linear regression coefficients are multiplied by the coefficients for all parent

groups to provide a single rank-ordered list of the potentially significant

evaluation measures.

7. Replace potentially significant evaluation measures with utility functions. In

 priority order, perform iterative substitutions. This is done by:

8. Set iteration counter $m = 1$

9. Select the remaining $v_i(x_i)$ from the list of potentially significant evaluation

 measures with the greatest coefficients from the RSM model.

10. Elicit $u_i(x_i)$ for this evaluation measure.

11. Determine $\hat{U}_j^{(m)} \forall j$ by substituting the $u_i(x_i)$ from the previous step.

12. If $m = 1$ or if either stopping criteria is not met, increment $m = m + 1$ and return to step b. Otherwise stop and $E\left[\hat{U}_*^{(m)}\right] \approx E\left[U_*\right]$. The stopping criteria are:

    a. All significant evaluation measures, as determined by RSM, have been converted from $v_i(x_i)$ to $u_i(x_i)$.

    b. $E\left[\hat{U}_j^{(m)}\right]$ is sufficiently close to $E\left[U_j\right]$.

13. The hybrid model is now used as would a standard utility model to:

    a. Select the best alternative, $\hat{U}_*^{(m)}$, from an expected value standpoint, and

    b. Perform a sensitivity analysis.

5 Demonstration of Value-Utility Hybridization

Klimack (2002b) provides an example of the employment of the hybrid value-utility approach. An illustrative summary is provided here. Klimack, Bassham, and Bauer (2002a) applied decision analysis to an Automatic Target Recognition (ATR) problem. This application of decision analysis included eliciting both single dimensional value and utility functions from an expert in the field. This provided an opportunity to employ the Value-Utility Hybridization methodology.

5.1 Background.

Automatic Target Recognition (ATR) is a processing problem where an image is examined to identify or otherwise characterize targets. All ATR classification systems (CSs) have a number of desirable traits that may be used as evaluation measures when selecting from among candidate systems. In general, these measures have not applied in total, but specific measures have been selected when considering alternatives for a specific program. A decision analysis approach was employed. A value model was constructed. The value hierarchy is presented in Figure 13, and captures the motivators important to the decision maker. Besides the values, the problem is affected by the two mission profiles (combat identification [CID] and intelligence/surveillance/reconnaissance [ISR]) under which the ATR CS would be employed. Additionally there are three alternatives under consideration. Employing single dimensional value functions and single dimensional utility functions produced differing results, indicating that the problem was sensitive to preference function construction.

5.2 Employing RSM to Prioritize Model Parameters

The ATR DA problem had 23 evaluation measures, 23 corresponding weights, two mission profiles (combat identification [CID] and intelligence/surveillance/reconnaissance [ISR]), and three alternatives. This was a moderately large number of parameters. Weights were perturbed using the procedure provided by Kirkwood (1997b: 83 – 84) as implemented by Bauer,
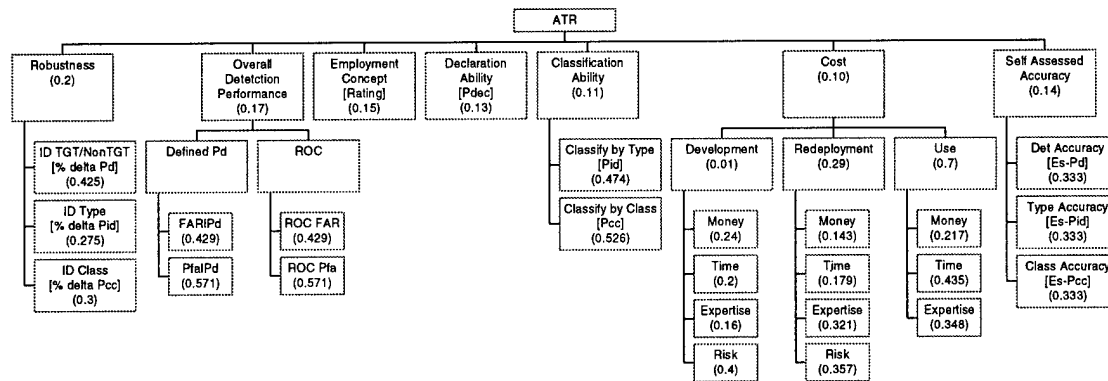
ATR

Robustness (0.2) | Overall Detetction Performance (0.17) | Employment Concept [Rating] (0.15) | Declaration Ability [Pdec] (0.13) | Classification Ability (0.11) | Cost (0.10) | Self Assessed Accuracy (0.14)

ID TGT/NonTGT [% delta Pd] (0.425) | Defined Pd | ROC

ID Type [% delta Pid] (0.275) | FARlPd (0.429) | ROC FAR (0.429)

ID Class [% delta Pcc] (0.3) | PfalPd (0.571) | ROC Pfa (0.571)

Classify by Type [Pid] (0.474) | Development (0.01) | Redeployment (0.29) | Use (0.7) | Det Accuracy [Es-Pd] (0.333)

Classify by Class [Pcc] (0.526) | Money (0.24) | Money (0.143) | Money (0.217) | Type Accuracy [Es-Pid] (0.333)

Time (0.2) | Time (0.179) | Time (0.435) | Class Accuracy [Es-Pcc] (0.333)

Expertise (0.16) | Expertise (0.321) | Expertise (0.348)

Risk (0.4) | Risk (0.357)

Figure 13. The Value Hierarchy for the ATR System. The dashed line under Overall Detection Performance indicates that either the Defined P_d or the ROC measures are employed. The parenthetical numbers indicate the relative weights for a value, within the parent value.

Parnell, and Meyers (1999: 168). The relative weight deemed most important, w_i, (generally the largest in magnitude) was varied by a factor of 0.1, or $w_i^+ = 1.1w_i$ and $w_i^- = 0.9w_i$. The remaining weights were adjusted to meet the summation constraint while retaining their relative ratios. The weights then become a single variable in the regression to find the response surface.

The remaining 25 variables (the 23 evaluation measure variables, the mission profile, and the alternative variables) are still a large number of parameters. To reduce the number to a more tractable amount, the group screening technique was employed. The first decomposition of the value hierarchy, the second row from the top, was used as the initial group screening set. RSM was employed on this groups to determine significant groups. RSM was then be employed separately for each decomposition of the significant value groups. The evaluation measure least square coefficients from the RSM analyses were then combined to produce a single list of significant evaluation measures. The list was prioritized based on least squares model coefficients. This prioritized list was used for construction of the hybrid value-utility model, which was then used to analyze the decision in typical fashion.

5.2.1 Initial Group Screening

For the ATR analysis, the grouping of the continuous variables was selected at the second level of the value hierarchy (see Figure 14). This provides seven groups:

Robustness,
Overall Detection Performance,

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Employment Concept,
Declaration Ability,
Classification Ability,
Cost, and
Self-Assessed Accuracy.

Counting the weight variable, there were eight continuous variables. There are also two categorical variables: the mission profile, with two levels, CID and ISR; and the alternative, with three levels, ATR 33, ATR 55, and ATR 89. The design was developed in the JMP® software package, which recommended a L36 Taguchi Design. Six center points were added to the L36 design, permitting the various combinations of profile and alternative to be examined. For this example only first order effects, without interactions, will be examined. As no interactions are under consideration, aliasing concerns are avoided.
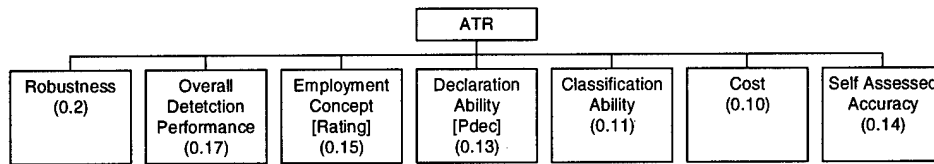

Figure 14. First Group Screening, Second Tier of Value Hierarchy.

The L36 design employs the continuous variables at a high or low setting, indicated by a plus or minus sign, respectively, in the pattern notation. The patterns for the matrix rows are shown in Table 1. For the continuous variables, the minus sign indicates the variable of interest is set to the lower level, a plus sign indicates it is set to the higher level. The number zero identifies center points. For the categorical variables, which are handled by JMP® with dummy variables, the minus, plus, and zero indicate different categories. For example, the lead minus sign in row 1 indicates that the robustness variable is at the lower setting. The order of the variables is as presented in the list above, then the profile and alternative variables. The data for these setting are presented below in Equations (50) through. Also shown in Table 1 are the results for each combination (row), $\hat{U}$.

Table 1. Initial Group Screening Design and Response for ATR Problem.

Row	Pattern	$\hat{U}$	Row	Pattern	$\hat{U}$	Row	Pattern	$\hat{U}$
1	----------	0.278	15	+-++-+---+	0.627	29	---+++-++0	0.360
2	---------0	0.302	16	++-++-+---	0.610	30	---+++-+++	0.376
3	---------+	0.292	17	++-++-+--0	0.623	31	+---+++-+-	0.524
4	+-+---+++-	0.535	18	++-++-+--+	0.602	32	+---+++-+0	0.515
5	+-+---+++0	0.505	19	+++-++-+--	0.772	33	+---+++-++	0.531
6	+-+---++++	0.506	20	+++-++-+-0	0.806	34	-+---+++--	0.414
7	++-+---++-	0.480	21	+++-++-+-+	0.795	35	-+---+++-0	0.433
8	++-+---++0	0.544	22	-+++-++-+-	0.512	36	-+---+++-+	0.414
9	++-+---+++	0.546	23	-+++-++-+0	0.565	37	00000000--	0.509
10	-++-+---+-	0.601	24	-+++-++-++	0.583	38	00000000-0	0.556
11	-++-+---+0	0.645	25	--+++-++--	0.500	39	00000000-+	0.525
12	-++-+---++	0.647	26	--+++-++-0	0.545	40	00000000+-	0.497
13	+-++-+----	0.574	27	--+++-++-+	0.541	41	00000000+0	0.531
14	+-++-+---0	0.622	28	---+++-++-	0.381	42	00000000++	0.497

To determine the response, the decision problem was modeled in the DPL® software package. The various evaluation measure levels, x_i, were passed to an Excel® spreadsheet where $v_i(x_i)$ was calculated. This intermediate result was used to determine $u_i(v_i(x_i))$ from

$$u_i(v_i(x_i)) = \frac{1 - e^{-v_i(x_i)/\rho_i}}{1 - e^{-1/\rho_i}} \qquad (49)$$

where

$$\rho_i = \begin{cases} 0.1 & \text{for pattern "+"} \\ \infty & \text{for pattern "0"} \\ -0.1 & \text{for pattern "-"} \end{cases} \qquad (50)$$

A zero indicates nominal setting, when $u_i(v_i(x_i)) = v_i(x_i)$.

The weight variable $\tilde{w}$ is a vector. The nominal value for the largest weight is that of the Robustness evaluation measure with a weight of 0.2. However no data was available for the robustness for identification of class. Because of this, the next largest evaluation measure weight was selected. This was the Overall Detection Performance evaluation measure, denoted w_2, equal to 0.17. Perturbing this by 0.1, or $w_2 = w_2 \pm 0.1w_2$, and adjusting the remaining weights

produces two weight vectors as called for by the high and low pattern indicators. As with ρ_i, a zero indicates nominal setting, or the elicited weights, $\tilde{\mathbf{w}}^0$.

These vectors are

$$\tilde{\mathbf{w}} = \begin{cases} \tilde{\mathbf{w}}^+ & \text{for pattern "+"} \\ \tilde{\mathbf{w}}^0 & \text{for pattern "0"} \\ \tilde{\mathbf{w}}^- & \text{for pattern "-"} \end{cases} \tag{51}$$

where

$$\tilde{\mathbf{w}}^+ = \begin{bmatrix} 0.1959 \\ 0.187 \\ 0.1469 \\ 0.1273 \\ 0.1077 \\ 0.098 \\ 0.1371 \end{bmatrix}, \quad \tilde{\mathbf{w}}^0 = \begin{bmatrix} 0.2 \\ 0.17 \\ 0.15 \\ 0.13 \\ 0.11 \\ 0.1 \\ 0.14 \end{bmatrix}, \quad \tilde{\mathbf{w}}^- = \begin{bmatrix} 0.2041 \\ 0.153 \\ 0.1531 \\ 0.1327 \\ 0.1123 \\ 0.1020 \\ 0.1429 \end{bmatrix} \tag{52}$$

The categorical variables, which were handled by employing dummy variables, were set according to

$$\text{Profile} = \begin{cases} \text{ISR} & \text{for pattern "+"} \\ \text{CID} & \text{for pattern "-"} \end{cases} \tag{53}$$

and

$$\text{Alternative} = \begin{cases} \text{ATR 89} & \text{for pattern "+"} \\ \text{ATR 55} & \text{for pattern "0"} \\ \text{ATR 33} & \text{for pattern "-"} \end{cases} \tag{54}$$

The response variable, utility $\hat{U}$, is also presented in Table 1. A plot of actual versus predicted values is contained in Figure 15. The linear model fit well with $R^2 = 0.978$ and $R^2_{adj} = 0.970$. The P value was less than 0.0001. The parameters estimates and the significance information for the model are contained in Table 2. The Declaration Ability and Self-Assessment groups are clearly not significant. They may be dropped from consideration. The weights are also not significant, which agrees with the sensitivity analysis performed as part of the basic DA analysis of the problem, and will be eliminated from further consideration. The other groups are clearly significant. Note that the coefficient estimates for the least squares model agree with the significance information. The significant variables have coefficients that

are an order of magnitude larger than those found to be non-significant. The coefficients also provide a measure of importance. Repeating the model fit with these groups removed would provide the same information (although the p-values would change slightly because of changes in the number of degrees of freedom) and so is not required.
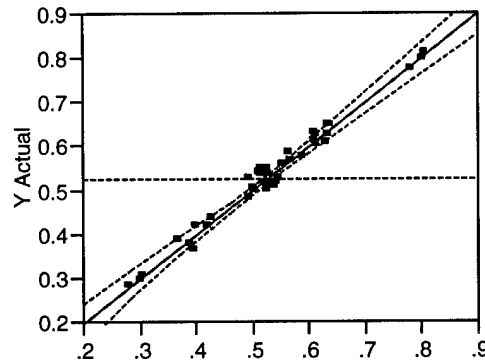

Figure 15. ATR Group Screening Model Actual versus Predicted Values.

The significance information in Table 2 may be used to prioritize the order of converting the single dimensional value functions to their utility equivalents. However the group screening has only eliminated five evaluation measures. Further, four groups have their significance levels indicated as equal. This fails to prioritize between eight evaluation measures. For further refinement the next step is to decompose the groups, to determine the significance of the group members.

5.2.2 Second Group Screening.

The candidate groups for decomposition and examination with RSM are: Robustness, Detection Performance, Classification Ability, and Cost. The Employment Concept criterion is not decomposable, so cannot be examined further. Employment Concept should be converted to utility in the hybrid model. (However of note is that all three candidates have equal scores for this criterion. If no additional candidates are introduced with a differing score for Employment Concept, and no sensitivity analysis is performed on the Employment Concept scores, it may be retained as a value function. Typically the sensitivity analysis restriction is onerous, but this

Table 2. ATR Group Screening Parameter Estimates.

Term	Estimate	Std Error	t Ratio	Prob > |t|
Intercept	0.529	0.00316	167.50	<.0001
Robustness	0.0647	0.00341	18.95	<.0001
Detection Performance	0.0577	0.00341	16.92	<.0001
Employment Concept	0.0738	0.00341	21.62	<.0001
Declaration Ability	0.00211	0.00341	0.62	0.541
Classification Ability	0.0456	0.00341	13.37	<.0001
Cost	0.0139	0.00341	4.09	0.0003
Self Assessment Accuracy	-0.00528	0.00341	-1.55	0.132
Weights	-0.00556	0.00341	-1.63	0.114
CID Profile	0.0109	0.00316	3.46	0.0016
ISR Profile	-0.0109	0.00316	-3.46	0.0016
ATR 33 Alternative	-0.0157	0.00447	-3.52	0.0014
ATR 55 Alternative	0.0104	0.00447	2.32	0.0274
ATR 89 Alternative	0.00536	0.00446	1.20	0.240

point might be valuable where many utility elicitations are involved.) Eliminated from the immediate requirement to convert from value to utility functions are Declaration Ability and Self-Assessment. Choosing the first subgroup to screen should be done with consideration as to the potential for eliminating criteria. The Cost measure provides this, but is complex with two sub-levels and eleven evaluation measures. For simplicity Robustness was chosen; other groups were addressed later. The Robustness portion of the value hierarchy is shown in Figure 16.
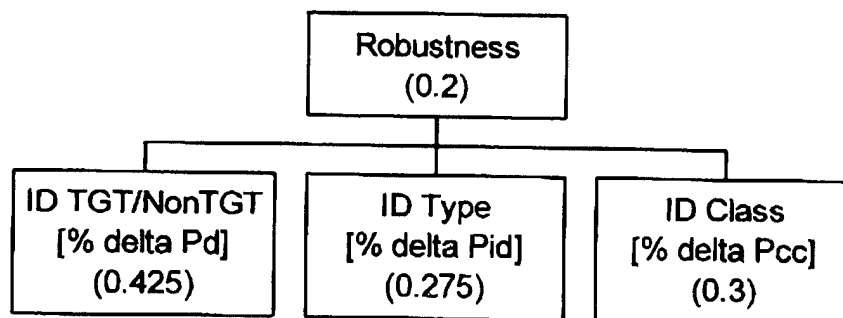

Figure 16. Second Group Screening, Robustness Portion of Value Hierarchy.

Robustness has three evaluation measures: Robustness in Detection (ΔP_d), Robustness in Typification (ΔP_{id}), and Robustness in Classification (ΔP_{cc}). The variables under consideration are the exponential constants for these measures. Refer to them as ρ_1, ρ_2, and ρ_3, respectively, for this portion of the analysis. They take on values as indicated in equation (50). The weights will be perturbed as above with the largest weight, that of Robustness in Detection, being perturbed ten percent about the nominal value. The equivalent of equation (52) becomes

$$\tilde{w}^+ = \begin{bmatrix} 0.4675 \\ 0.2547 \\ 0.2778 \end{bmatrix}, \quad \tilde{w}^0 = \begin{bmatrix} 0.425 \\ 0.275 \\ 0.3 \end{bmatrix}, \quad \tilde{w}^- = \begin{bmatrix} 0.3825 \\ 0.2953 \\ 0.3222 \end{bmatrix} \tag{55}$$

The profile and alternative information remained unchanged. Categorical data again restricted the possible experimental designs available. A Hunter L18 (Sall, 2001: 57) design was selected. This guarantees orthogonality and there are no aliasing concerns, as we do not investigate interactions here. The Robustness screening experimental design is illustrated in Table 3, along with the results. The pattern information indicates variables levels as explained above, with a variable order of Robustness in Detection, Robustness in Typification, and Robustness in Classification, weight, profile, and alternative. Analysis of these data provides the coefficient estimates in Table 4.

As can be seen from the data in Table 4, Robustness in Detection and Robustness in Typification are significant while Robustness in Classification is not significant. The first two evaluation measures are candidates for conversion from $v_j(x_j)$ to $u_j(x_j)$ while the last is excluded from consideration. The mission profile is not significant. Any other result would be surprising, as the input data for robustness was not conditioned on mission profile. The alternatives are not significant. This indicates that the most desired alternative changes over the range of the perturbation introduced in this analysis.

5.2.3 Third Group Screening.

Turning to consider cost, analysis proceeded in similar fashion. The least squares fit of the data is provided in Table 5. The Cost Use Time was highly significant, but the model fit was mediocre, with $R^2 = 0.657$ and $R^2_{adj} = 0.523$. Because of the fit, Cost Redeployment Time, Cost Use Experience, and Cost Redeployment Risk were retained in the list of significant variables.

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Table 3. Robustness Screening Experimental Design and Response for ATR Problem.

Row	Pattern	$\hat{U}$	Row	Pattern	$\hat{U}$	Row	Pattern	$\hat{U}$
1	---++-	0.0289	9	--+-+0	0.331	17	-----+	0.324
2	+++---	0.949	10	++-+-0	0.995	18	++++++	0.987
3	-++---	0.577	11	-+-+-0	0.534	19	000011	0.618
4	+--++-	0.745	12	+-+-+0	0.708	20	000012	0.671
5	-+--+-	0.577	13	-+++++	0.526	21	000013	0.584
6	+-++--	0.745	14	+----+	0.701	22	000021	0.618
7	--++-0	0.286	15	-----+	0.324	23	000022	0.671
8	++---+0	0.996	16	++++++	0.987	24	000023	0.584

Table 4. ATR Robustness Screening Parameter Estimates.

Term	Estimate	Std Error	t Ratio	Prob > |t|
Intercept	0.628	0.0127	49.38	<.0001
Robustness in Detection	0.226	0.0149	15.10	<.0001
Robustness in Typification	0.140	0.0149	9.39	<.0001
Robustness in Classification	0.0113	0.0149	0.75	0.461
Weights(-1,1)	-0.0214	0.0149	-1.43	0.171
Profile[CID]	0.0109	0.0129	0.84	0.411
Profile[ISR]	-0.0109	0.0129	-0.84	0.411
Alternative[ATR 33]	-0.0207	0.0180	-1.15	0.267
Alternative[ATR 55]	0.0213	0.0180	1.18	0.254
Alternative[ATR 89]	-0.000597	0.0180	-0.03	0.974

Table 5. ATR Cost Screening Parameter Estimates.

Term	Estimate	Std Error	t Ratio	Prob > |t|
Intercept	0.590	0.0140	42.27	<.0001
Development Money(-1,1)	-0.00393	0.0143	-0.28	0.785
Development Time(-1,1)	0.00171	0.0143	0.12	0.906
Development Expertise(-1,1)	0.0131	0.0143	0.91	0.368
Development Risk(-1,1)	-0.00183	0.0144	-0.13	0.899
Redeployment Money(-1,1)	0.0101	0.0144	0.70	0.489
Redeployment Time(-1,1)	0.0239	0.0144	1.66	0.106
Redeployment Expertise(-1,1)	0.0170	0.0144	1.18	0.244
Redeployment Risk(-1,1)	0.0231	0.0144	1.60	0.118
Use Money(-1,1)	0.0169	0.0144	1.17	0.249
Use Time(-1,1)	0.0569	0.0143	3.97	0.0003
Use Expertise(-1,1)	0.0233	0.0144	1.62	0.115
Weights(-1,1)	0.0261	0.0144	1.81	0.0787
ATR[ATR 33]	0.0805	0.0196	4.10	0.0002
ATR[ATR 55]	-0.114	0.0196	-5.83	<.0001
ATR[ATR 89]	0.0340	0.0196	1.73	0.0924

5.2.4 Fourth Group Screening

The Detection Performance taxon was next examined. This decomposed into two evaluation measures, False Alarm Rate given a Probability of Detection and the Probability of False Alarm given a Probability of Detection. (Alternatively, it could have been structured on the Required Operating Curve False Alarm Rate and the Required Operating Curve Probability of False Alarm.) Analysis proceeded as above. The least square variable significance and parameter estimates are in Table 6. As seen in Table 6, both the False Alarm Rate and the Probability of False Alarm were found to be significant.

Table 6. ATR Detection Performance Screening Parameter Estimates.

Term	Estimate	Std Error	t Ratio	Prob > ltl
Intercept	0.471	0.0247	19.02	<.0001
FAR(-1,1)	0.1293	0.0290	4.46	0.0003
Pfa(-1,1)	0.214	0.0290	7.38	<.0001
w(-1,1)	-0.0207	0.0290	-0.71	0.486
Profile[CID]	0.00384	0.0250	0.15	0.880
Profile[ISR]	-0.00384	0.0250	-0.15	0.880
ATR[ATR 33]	-0.195	0.0350	-5.58	<.0001
ATR[ATR 55]	0.0653	0.0350	1.86	0.0796
ATR[ATR 89]	0.130	0.0350	3.72	0.0017

5.2.5 Fifth Group Screening

Next Classification Ability was analyzed. The coefficients and significance statistics are contained in Table 7. Both Classify by Type and Classify by Class evaluation measures are highly significant.

5.2.6 Sixth Group Screening

Finally the Self Assessed Accuracy category was considered. The data and both value and utility functions are identical for the three evaluation measures in this category. The input data are all unity, providing value and utility of zero, as this evaluation measure was not part of the ATR CS testing. So testing this category as above was useless, as all combinations of weights and utility exponential values produce a utility of zero. Clearly the Self Assessed Accuracy measures are not significant.

Table 7. ATR Classification Ability Screening Parameter Estimates.

Term	Estimate	Std Error	t Ratio	Prob > |t|
Intercept	0.534	0.0204	26.12	<.0001
Type(-1,1)	0.215	0.0240	8.97	<.0001
Class(-1,1)	0.267	0.0239	11.2	<.0001
w(-1,1)	0.00705	0.0239	0.29	0.772
Profile[CID]	0.0649	0.0207	3.14	0.0060
Profile[ISR]	-0.0649	0.0207	-3.14	0.0060
ATR[ATR 33]	0.00477	0.0289	0.16	0.871
ATR[ATR 55]	0.0129	0.0289	0.45	0.661
ATR[ATR 89]	-0.0177	0.0289	-0.61	0.550

5.2.7 Prioritization of Evaluation Measures

The ordered coefficients are contained in Table 8. They were determined by multiplying coefficients by the coefficient of parent groups to provide a global comparison. These provide a reasonable order in which single dimensional value functions should be converted to single dimensional utility functions in the hybrid model.

The value hierarchy reduced to those evaluation measures that were determined to be significant for $\alpha = 0.10$ during the screening process is depicted in Figure 17. Examining only these evaluation measures produces the subset of model coefficients listed in Table 9. The entries of Table 9 are identical to the top eleven entries of Table 8 except that Declaration Ability and Classification Robustness do not appear in Table 9. These evaluation measures were not found to be significant during the screening. They are in the eighth and tenth positions of Table 8. This indicates that employing the significance as a screening tool is reasonable.

5.3 Employment of the Hybrid Value-Utility Model

The hybrid utility algorithm was applied, using the priority sequence of variables presented in Table 9. Iteration zero employs the value model. Each succeeding iteration provides a hybrid model with the next highest priority evaluation measure value function replaced by the utility function. The results for the mission CID profile are presented in Table 10. The pure utility model results are also provided.

Table 8. Rank Ordered, by Absolute Value, Model Coefficients.

| Evaluation Measure	Coefficient
Employment Concept	0.0738
Detection Robustness	0.0146
Probability of False Alarm	0.0123
Classification by Class	0.0122
Classification by Type	0.00980
Identification Robustness	0.00907
False Alarm Rate	0.00746
Declaration Ability	0.00211
Cost Use Time	0.000794
Classification Robustness	0.000729
Cost Redeployment Time	0.000334
Cost Use Expertise	0.000324
Cost Redeployment Risk	0.000322
Cost Redeployment Expertise	0.000238
Cost Use Money	0.000235
Cost Development Expertise	0.000183
Redeployment Money	0.000141
Cost Development Money	-5.48×10^{-5}
Cost Development Risk	-2.55×10^{-5}
Cost Development Time	2.38×10^{-5}
Es Pd	0
Es Pid	0
Es Pcc	0
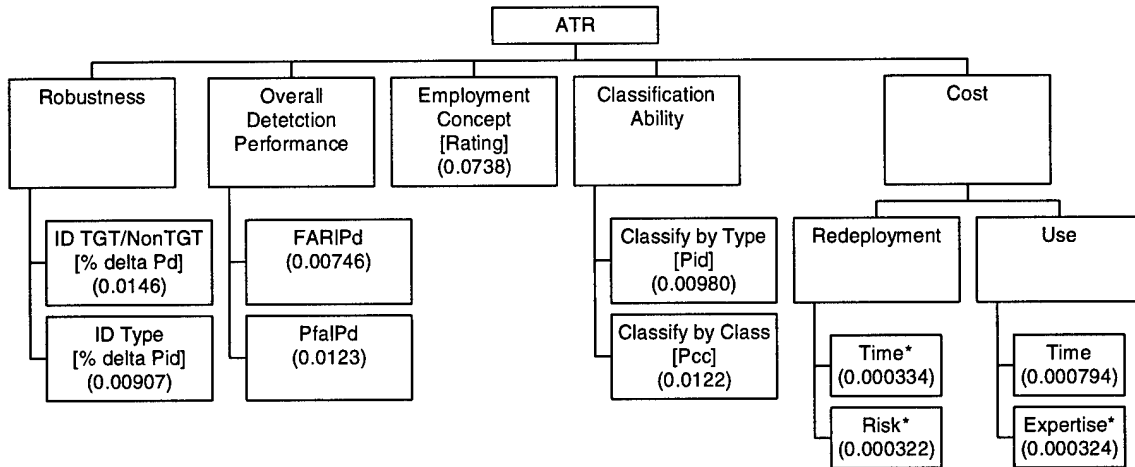
Figure 17. Value Hierarchy Reduced to Significant Taxa. The parenthetical numbers indicate the estimated coefficients. Taxa significant at alpha equal to 0.1 and not when alpha equal 0.05 are indicated with an asterisk.

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Table 9. Significant Rank Ordered Model Coefficients. Those significant at an alpha equal to 0.1 and not when alpha equal 0.05 are indicated with an asterisk.

Evaluation Measure	Coefficient	Priority
Concept	0.0738	1
Detection Robustness	0.0146	2
Probability of False Alarm	0.0123	3
Classification by Class	0.0122	4
Classification by Type	0.00980	5
Identification Robustness	0.00907	6
False Alarm Rate	0.00746	7
Cost Use Time	0.000794	8
Cost Redeployment Time*	0.000334	9
Cost Use Expertise*	0.000324	10
Cost Redeployment Risk*	0.000322	11

The hybrid utility model results are shown graphically in Figure 18 for the ATR 33 alternative. Other alternatives behaved similarly and are omitted for clarity. All three alternatives converge towards the utility model after an initial period of instability. This is more readily observed in Figure 19 through Figure 22, where the differences between iterative hybrid model scores and the distance from the true utility model for the two mission profiles are illustrated. The decreasing differences between iterations and the decreasing distance from the utility model are obvious. The latter measure is a measure of the error of the hybrid model compared to the true utility model. After the initial period this error remains small.

To understand the behavior of the hybrid model as it is modified during each iteration, we examine the initial step for the ATR 33 alternative (see Table 10 and Figure 18). The complete value model had an overall value of 0.509 for this alternative. When the Employment Concept was converted to the single dimensional utility function, the hybrid utility became 0.464. The Employment Concept rating for the ATR 33 alternative (indeed, all alternatives) was poorly defined. The corresponding single dimensional value is 0.4. The single dimensional utility for the poorly defined rating is 0.1, so the difference between the preference functions is 0.3. When this difference is multiplied by the relative weight for the Employment Concept evaluation measure, 0.15, the result is 0.45, the difference between the results obtained in the first iteration compared to the starting value model.

Table 10. CID Profile Value, Utility, and Hybrid Utility Results, Significant Evaluation Measures. Those significant at an alpha equal to 0.1 and not when alpha equal 0.05 are indicated with an asterisk.

|	Notes	ATR 33	ATR 55	ATR 89
Goal	Utility Model	0.572	0.507	0.518
Iteration				
0	Value Model	0.509	0.556	0.525
1	Employment Concept	0.464	0.511	0.480
2	Detection Robustness	0.455	0.471	0.442
3	Probability of False Alarm	0.529	0.500	0.512
4	Classification by Class	0.536	0.509	0.524
5	Classification by Type	0.538	0.511	0.528
6	Identification Robustness	0.529	0.505	0.509
7	False Alarm Rate	0.569	0.506	0.510
8	Cost Use Time	0.569	0.506	0.513
9	Cost Redeployment Time*	0.570	0.507	0.513
10	Cost Use Expertise*	0.570	0.507	0.513
11	Cost Redeployment Risk*	0.570	0.504	0.516
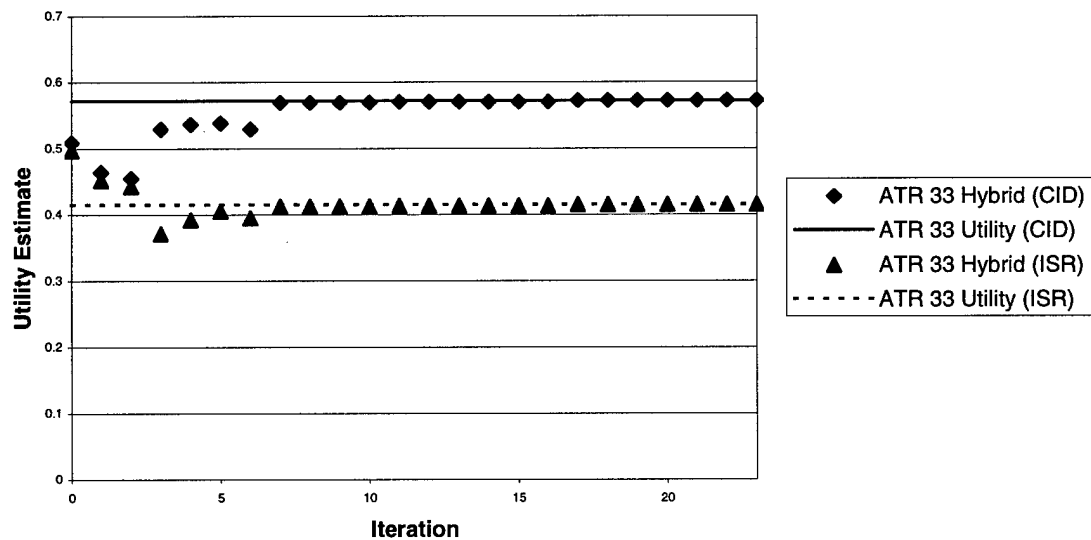
Figure 18. ATR Hybrid Utility Iterations for ATR 33.
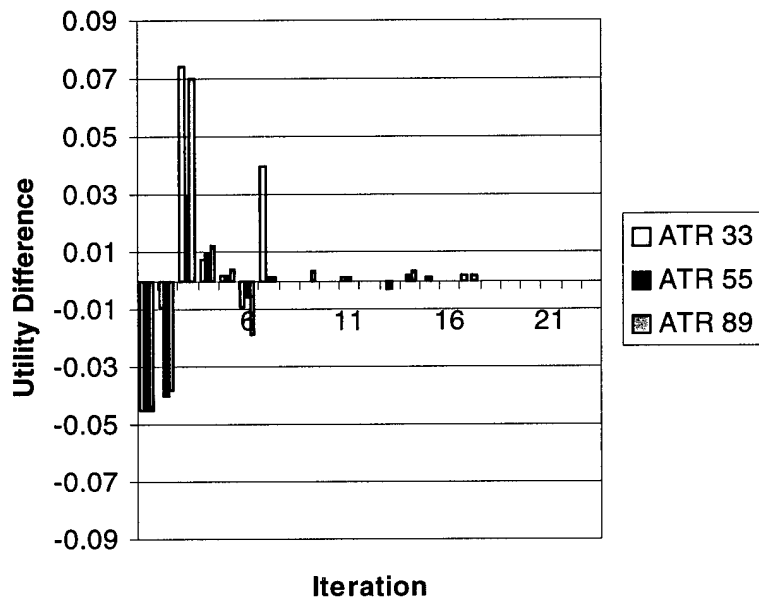
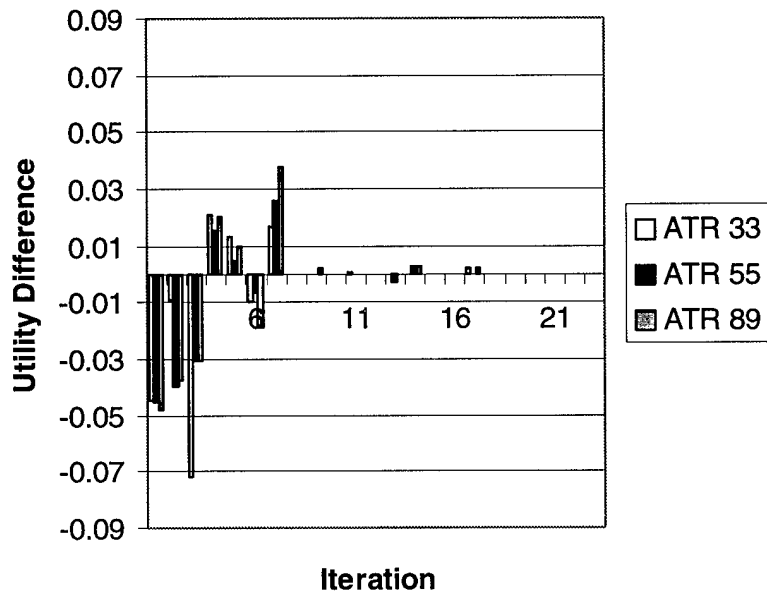Figure 19. Hybrid Utility Model Difference Between Successive Iterations, CID Profile.

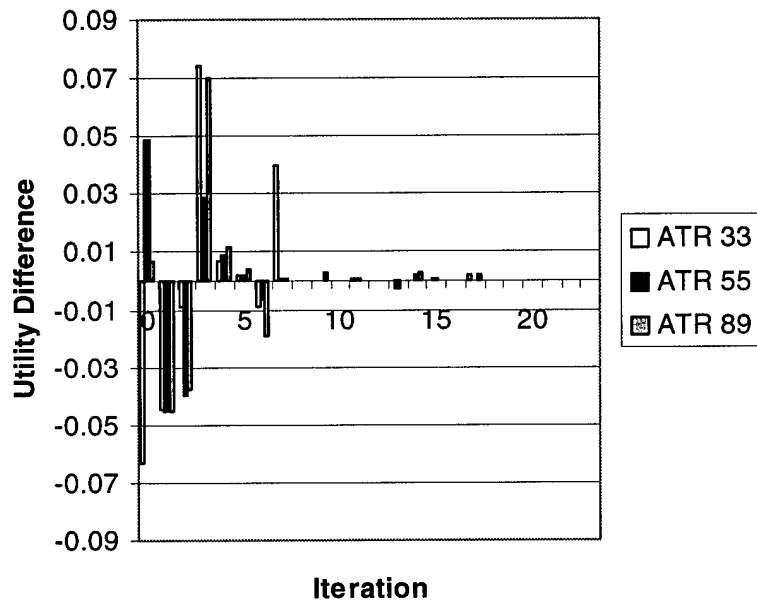Figure 20. Hybrid Utility Model Difference Between Successive Iterations, ISR Profile.

Figure 21. Hybrid Utility Model Difference Between Iteration and True Utility Model, CID Profile.
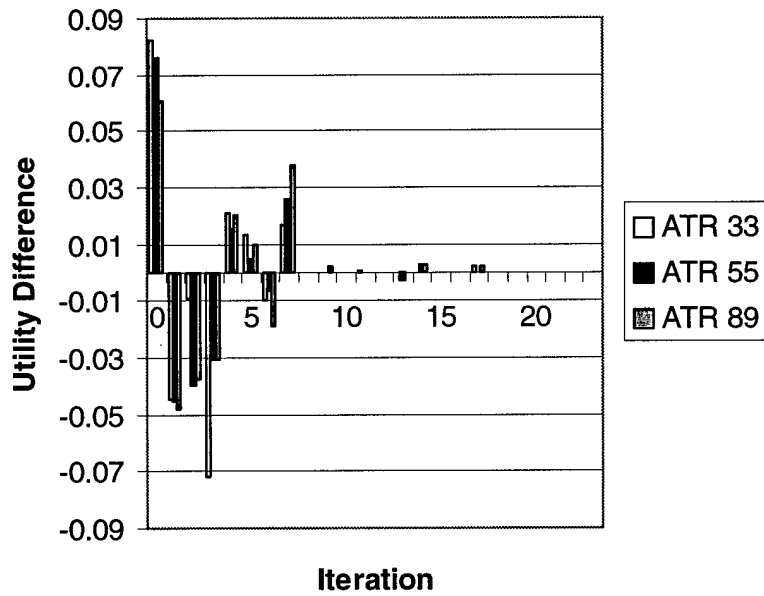

Figure 22. Hybrid Utility Model Difference Between Iteration and True Utility Model, ISR Profile.

It is interesting to note in Figure 18 that ATR 33 converges to the true utility model strictly from below for the CID profile, although the direction of movement was not identical for each iteration, while under the ISR case ATR 33 oscillated about the true utility value. In no case was the movement strictly towards the true utility, as would be expected if the decision maker was strictly relatively risk averse (seeking).

Once the significant evaluation measures, at $\alpha = 0.05$, were converted from value to utility functions the hybrid function provided estimates that differed at most 0.003 from the pure utility function result. The evaluation measures that became significant between $\alpha = 0.05$ and $\alpha = 0.1$ provided no improved estimate.

We conclude that for the example the evaluation measures determined to be significant through response surface methodology techniques provided an adequate set of variables to permit creation of an accurate hybrid model. The hybrid model may be used to represent the utility model. The number of preference functions required to be elicited from the decision maker was reduced from 26 to eight for $\alpha = 0.05$.

Further, the rank ordering by regression coefficient was successful in providing successive hybrid estimates that in general approached the true utility value, each with generally reduced difference (δ_i) from the previous estimate.

6 Summary

The algorithm provides a hybrid value-utility model. The hybrid utility model, $\hat{U}$, provided an adequate representation of the true utility model, U. Elicitation of the hybrid utility model is more efficient with respect to decision maker participation than employing the strict utility model, especially in cases where there are a large number of evaluation measures or cases where several of the measures require an inordinate amount of time to elicit additional utility information. Such efficiency promotes use of decision analysis and encourages sound decision-making. Under conditions of uncertainty and risk, the value model may provide differing results than the utility model, as was demonstrated in the example, and so is not acceptable.

This algorithm is unique in several ways. Response surface methodology is used to determine when the value model is sensitive to the preference functions. Response surface methodology was used to examine the margins of an envelope of reasonable curves for $\hat{u}_i\left(v_i\left(x_i\right)\right)$. When the function failed to be significant, then the true function was assumed to be not significant for evaluation measure i.

While value and utility are differing constructs, prudent exploitation of their differences and similarities permits more efficient use of the decision maker's time. This approach emphasizes that the distinction between value and utility must not be ignored. But the lack of strategic differences for specific cases may be identified and exploited

Appendix A: Von Neumann Expected Utility Axioms

Others have rewritten the von Neumann and Morgenstern (1953) axioms for clarity. For their description, the presentation generally follows Biswas (1997: 17 – 18). Let E denote the outcome set, so $x \in E$ is an outcome. Let $X = (x_1, x_2, \cdots, x_n)$ be the vector of n possible discrete realizations from the outcome set, each with a respective probability of occurrence, p_i. Represent a lottery of these n outcomes with $L = (p_1 x_1, p_2 x_2, \cdots, p_n x_n)$. Preference relationships will be written for $x_i, x_j \in X$ as $x_i \succ x_j$ when x_i is preferred to x_j, and $x_i \sim x_j$ when a subject is indifferent between x_i and x_j. Similarly, to represent x_i is at least as preferred as x_j, we write $x_i \succsim x_j$. Without loss of generality, we will assume that $x_1 \succsim x_2 \succsim \cdots \succsim x_n$.

A.1 Axiom 1 (Ordering).

Preferences may be (totally) ordered between any two outcomes. This ordering is a total ordering. So for any (x_i, x_j) either $x_i \succsim x_j$ or $x_j \succsim x_i$. Further, $x_i \succsim x_j$ and $x_j \succsim x_k$ implies $x_i \succsim x_j \succsim x_k$.

A.2 Axiom 2 (Reduction of Compound Lotteries).

An individual is indifferent between a simple lottery L, (a lottery with outcomes that are fixed) and a compound lottery (a lottery with one or more outcomes that are also lotteries) with the same outcomes when the probabilities of the simple lottery are determined in accordance with standard manipulation of probabilities. If a lottery $L^{(m)}$ is defined as $L^{(m)} \equiv (p_1^m x_1, p_2^m x_2, \cdots, p_n^m x_n)$, where the superscripts indicate membership in a compound lottery m, it follows that $(q_1 L^{(1)}, q_2 L^{(2)}, \cdots, q_o L^{(o)}) \sim (p_1 x_1, p_2 x_2, \cdots, p_n x_n)$ where $p_i = q_i p_i^{(1)} + \cdots + q_o p_i^{(o)}$ and $\sim$ represents indifference.

For clarity, this axiom will be detailed for the case of two lotteries each with possible outcomes x_1 and x_2. These lotteries are $L^{(1)} = (p_1^{(1)} x_1, p_2^{(1)} x_2)$ and $L^{(2)} = (p_1^{(2)} x_1, p_2^{(2)} x_2)$. They are shown graphically in Figure 23. Then $(q_1 L^{(1)}, q_2 L^{(2)}, \cdots, q_o L^{(o)}) \sim (p_1 x_1, p_2 x_2, \cdots, p_n x_n)$

becomes $\left(q_1 L^{(1)}, q_2 L^{(2)}\right) \sim \left(p_1 x_1, p_2 x_2\right)$ where $p_i = q_i p_i^{(1)} + \cdots + q_o p_i^{(o)}$ or $p_1 = q_1 p_1^{(1)} + q_2 p_1^{(2)}$ and

$p_2 = q_i p_2^{(1)} + q_2 p_2^{(2)}$. So an individual would be indifferent between the choice of the compound

lottery or the corresponding outcomes with the associated reduced probabilities,

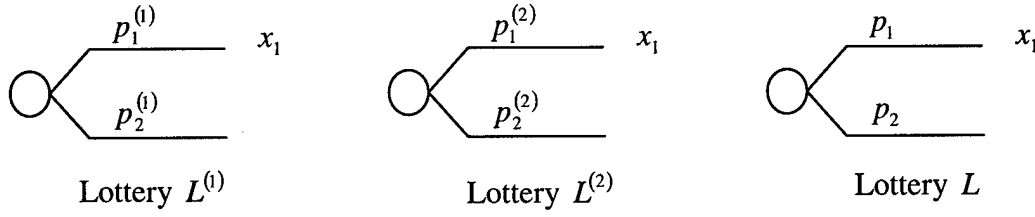$\left(q_1 L^{(1)}, q_2 L^{(2)}\right) \sim \left(p_1 x_1, p_2 x_2\right)$.

Figure 23. Lotteries $L^{(1)}$ $L^{(2)}$, and L.

A.3   Axiom 3 (Continuity).

An individual will be indifferent between some x_i and a lottery with outcomes x_1 and

x_n. That is, there exists some probability q_i such that $x_i \sim \left(q_i x_1, 0 x_2, 0 x_2, \cdots, (1 - q_i) x_n\right)$. Define

this lottery $L^{\tilde{x_i}} = \left(q_i x_1, (1 - q_i) x_n\right)$.

A.4   Axiom 4 (Substitutability).

In any lottery, $L^{\tilde{x_i}}$ may be substituted for x_i.

A.5   Axiom 5 (Transitive Preference over Lotteries).

Preference ordering is transitive over lotteries. That is $L^{(i)} \succsim L^{(j)}$ and $L^{(j)} \succsim L^{(k)}$ implies

$L^{(i)} \succsim L^{(j)} \succsim L^{(k)}$.

A.6   Axiom 6 (Monotonicity).

An individual prefers a lottery $L = \left(px_1, (1-p)x_2 \right)$ to a distinct lottery

$L' = \left(p'x_1, (1-p')x_2 \right)$ if and only if $p > p'$. Similarly, an individual is indifferent to a lottery

$L = \left(px_1, (1-p)x_2 \right)$ when compared to another lottery $L' = \left(p'x_1, (1-p')x_2 \right)$ if and only if

$p = p'$.

Appendix B: Measurable Value Function Axioms

Krantz, Luce, Suppes, and Tversky (1971) provide a set of axioms that establish when relationships between objects may be defined as an algebraic difference structure. (Other sources of the axioms, or alternative formulations of them, are [Fishburn, 1970], [Suppes and Winet, 1955], and [Debrau, 1960].) When these axioms hold, a real-valued function exists that provides cardinal preference information about the objects.

B.1   Definition (Algebraic Difference Structure).

The notation ab represents the increase in preference resultant from receiving object b in lieu of object a. Suppose A is a nonempty set and $\succeq$ is a binary relation on $A \times A$. The pair $\langle A \times A, \succeq \rangle$ is an algebraic difference structure if and only if for all $a,b,c,d,e,f \in A$ and all sequences $a_1, a_2, \cdots, a_i, \cdots \in A$, the following axioms hold true:

B.2   Axiom 1.

$\langle A \times A, \succsim \rangle$ is of weak order. A relation, R, on a set, A, is of weak order when it is transitive and strongly complete. Transitivity exists when aRb and bRc implies aRc for all $a,b,c \in A$. A relation is strongly complete on a set when aRb or bRa holds for all $a,b \in A$, including when $a = b$. (Roberts, 1979: 20 and 29)

B.3   Axiom 2.

If $ab \succsim cd$, then $dc \succsim ba$.

B.4   Axiom 3.

If $ab \succsim de$ and $bc \succsim ef$, then $ac \succsim df$.

B.5   Axiom 4.

If $ab \succsim cd \succsim aa$, then there exist $e, f \in A$, such that $ae \sim cd \sim fb$. (The requirement that $ab \succsim cd$ be $\succsim aa$ ensures that ab is increasing. This is required so that e, f be members of the interval $[a, b]$. When $e, f \in \overline{ab}$ then it must be true that $e, f \in A$.)

B.6   Axiom 5.

If $a_1, a_2, \cdots, a_i, \cdots$ is a strictly bounded standard sequence $a_{i+1}a_i \sim a_2 a_1$ for every a_i, a_{i+1} in the sequence, not $a_2 a_1 \sim a_1 a_1$; and there exists $e, f \in A$ such that $ef \succ a_i a_1 \succ fe$ for all a_i in the sequence), then it is finite. (Krantz, Luce, Suppes, and Tversky, 1971: 151)

Krantz et al show that if $\langle A \times A, \succsim \rangle$ is an algebraic difference structure, then there exists a real-valued function v on A such that, for all $a, b, c, d \in A$, $ab \succsim cd$ iff $v(a) - v(b) \geq v(c) - v(d)$ and v is unique up to a linear transformation. Methods of elicitation of measurable value functions are described by Fishburn (1967 and 1976); Johnson and Huber (1977); Kneppereth, Gustafson, Leifer, and Johnson (1974); Dyer and Sarin, (1979 and 1982); Farquhar and Keller, (1989); and Camacho (1980).

Appendix C: List of Abbreviations

A	
ATR	Automatic Target Recognition
C	
CID	Combat Identification
CS	Classification Systems
D	
DA	Decision Analysis
DSA	Dependent sensitivity analysis
I	
ISR	Intelligence/Surveillance/Reconnaissance
O	
ORCEN	Operations Research Center
R	
ROSA	Rank Order Stability Analysis
RSM	Response Surface Methodology

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